



# **PBW 620 Advanced Soil Mechanics**

# **PBW 584 Applied Soil Mechanics**

**Public Works Department**

**MSc. Degree**

**Spring Semester**

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**Soil Mechanics and Foundations Research Lab**

**Faculty of Engineering- Cairo University**

Lecture Three



# **ANALYTICAL APPROACHES TO CALCULATE EARTH PRESSURE AND BASE STABILITY**

# Lecture Outline

- Earth Pressure at rest
- Limiting States of Earth Pressure
- Analysis assuming geostatic conditions
- Analysis using limit equilibrium

# Earth Pressure At Rest

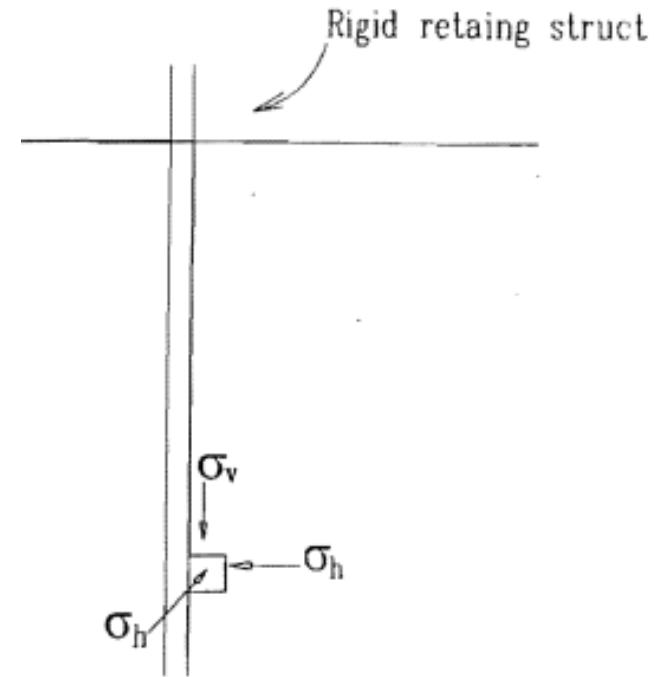
$$\epsilon_h = 1/E (\sigma_h - \mu (\sigma_v + \sigma_h))$$

$$\epsilon_h = 0$$

$$\sigma_h = \mu (\sigma_v + \sigma_h)$$

$$\sigma_h (1 - \mu) = \mu \sigma_v$$

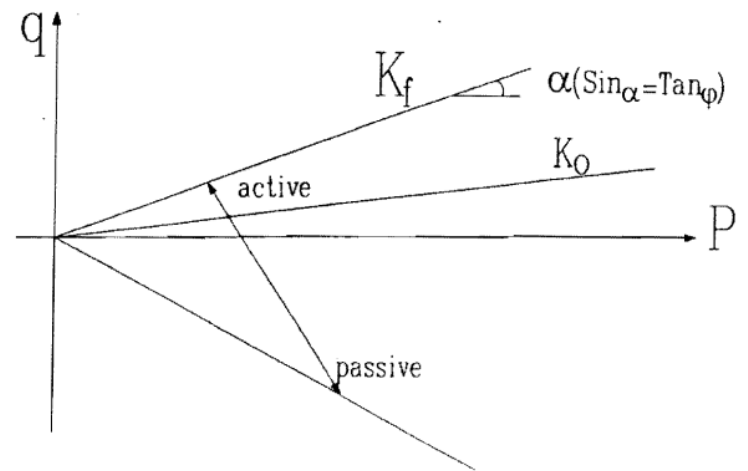
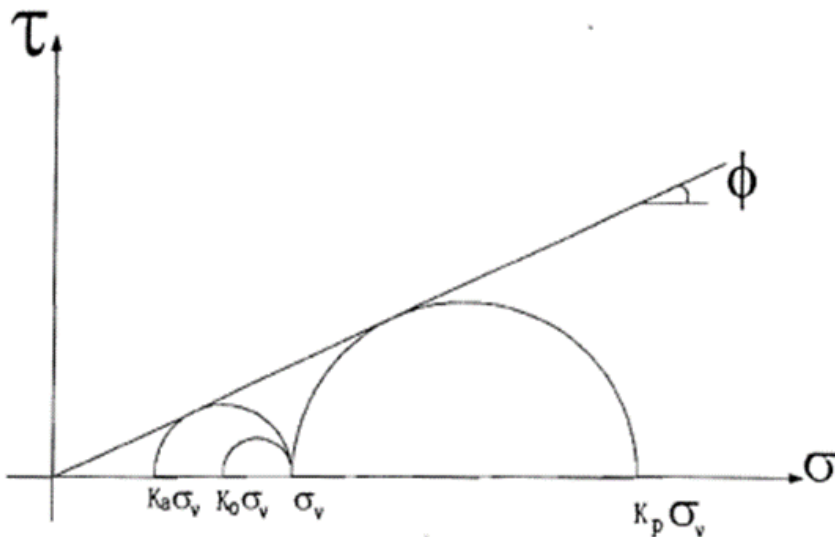
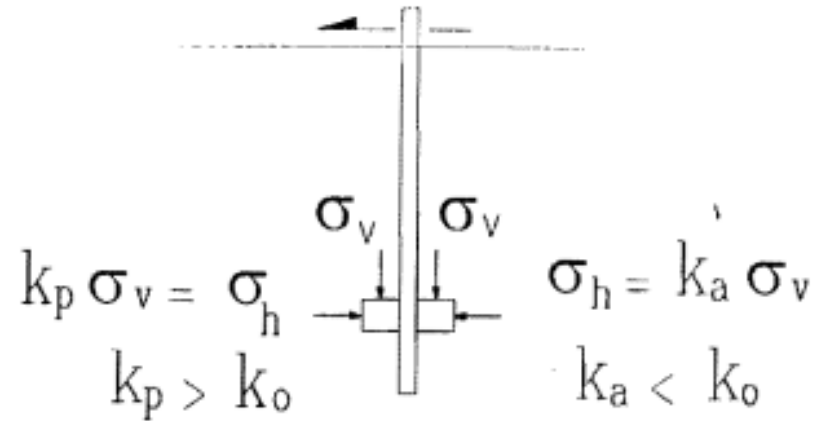
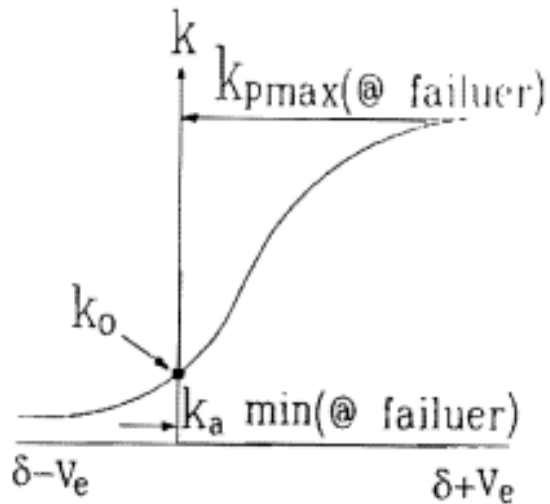
$$\frac{\sigma_h}{\sigma_v} = k_0 = \frac{\mu}{1 - \mu}$$



N.B \* Max value of  $\mu = 0.5$  (in compressible material ; water)

$$K_0 = \frac{0.5}{1 - 0.5} = 1 \text{ (Hydrostatic condition)}$$

# Active and Passive States of EP



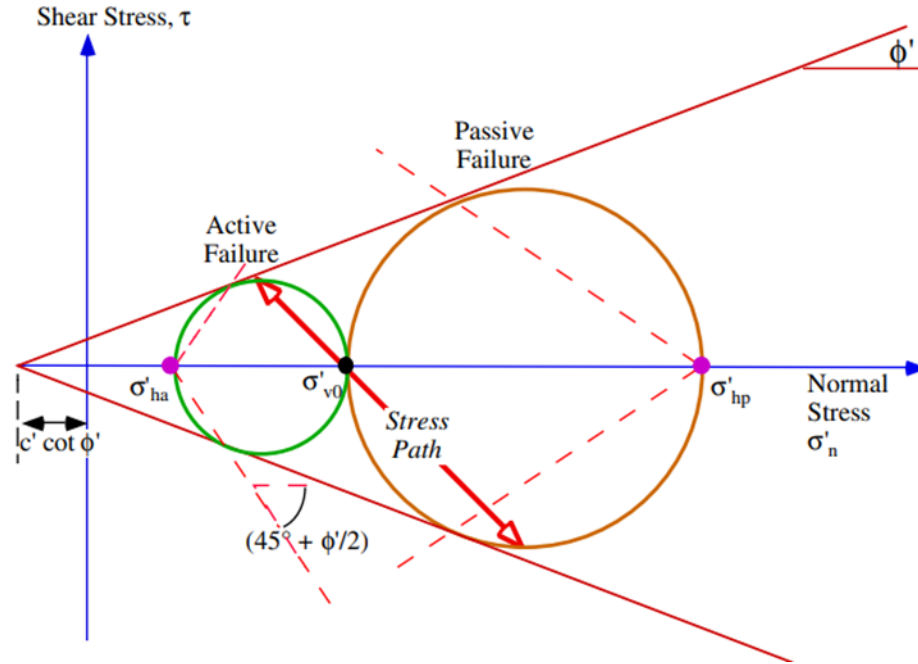
# Geostatic Conditions

$$\sigma_v = \sigma_1, \quad \sigma_h = \sigma_3$$

## Assumptions

- Horizontal Ground Surface
- Smooth Vertical Wall
- Soil is infinite
- Soil is cohesionless
- Soil is Homogenous
- Plane Strain Conditions
- Rigid Soil (Ignore deformations)

# Active and Passive EP coefficient



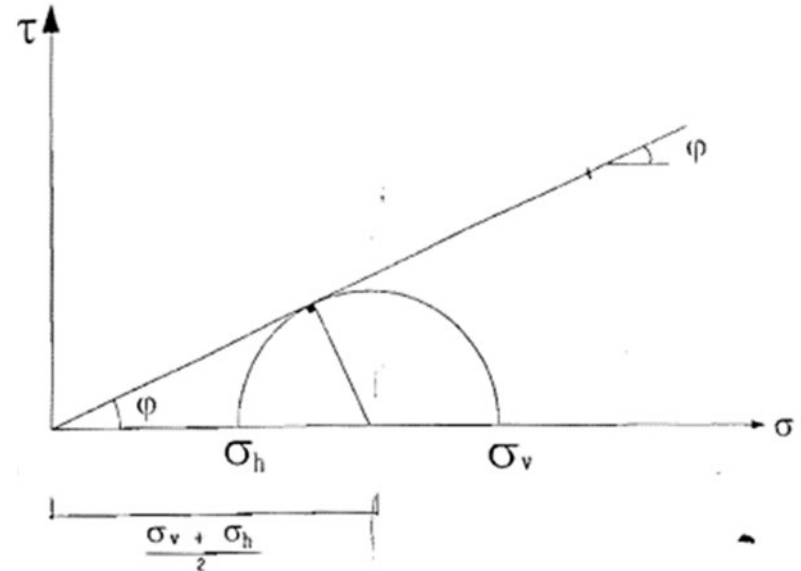
$$\sin \phi = \frac{(\sigma_1 - \sigma_3) / 2}{(\sigma_1 + \sigma_3) / 2}$$

$$\sigma_1 - \sigma_3 = \sin \phi \sigma_1 + \sin \phi \sigma_3$$

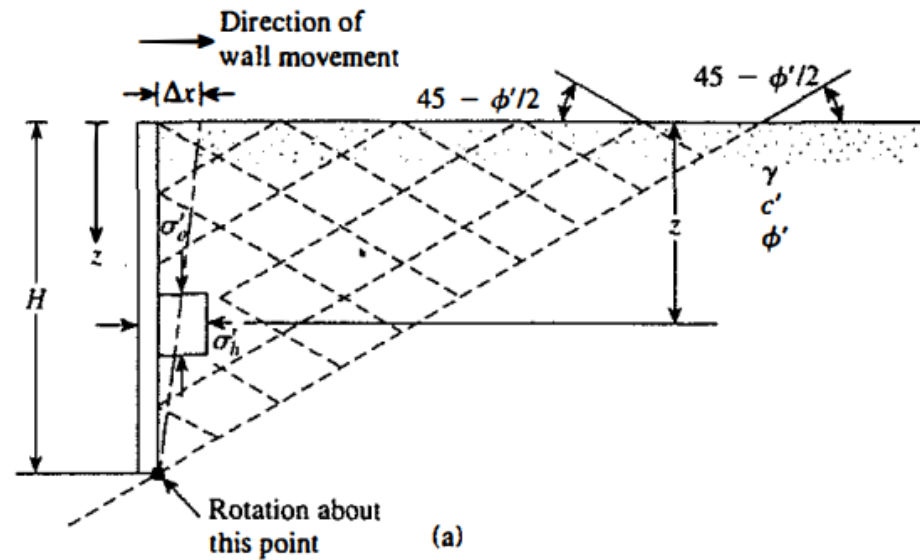
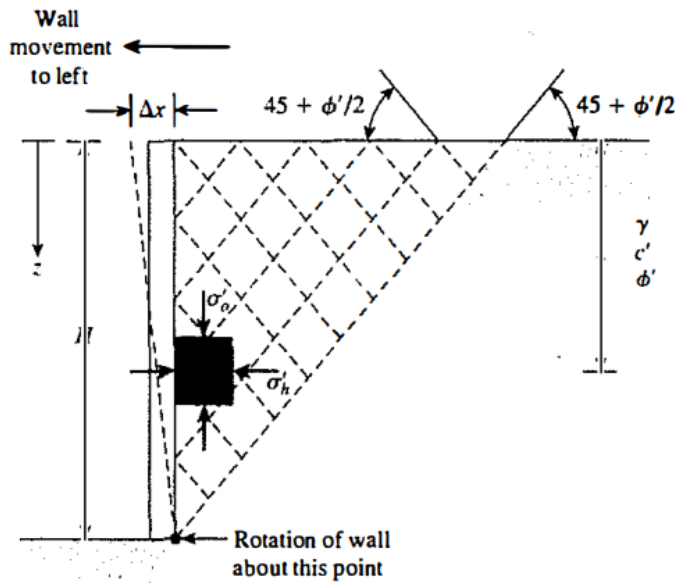
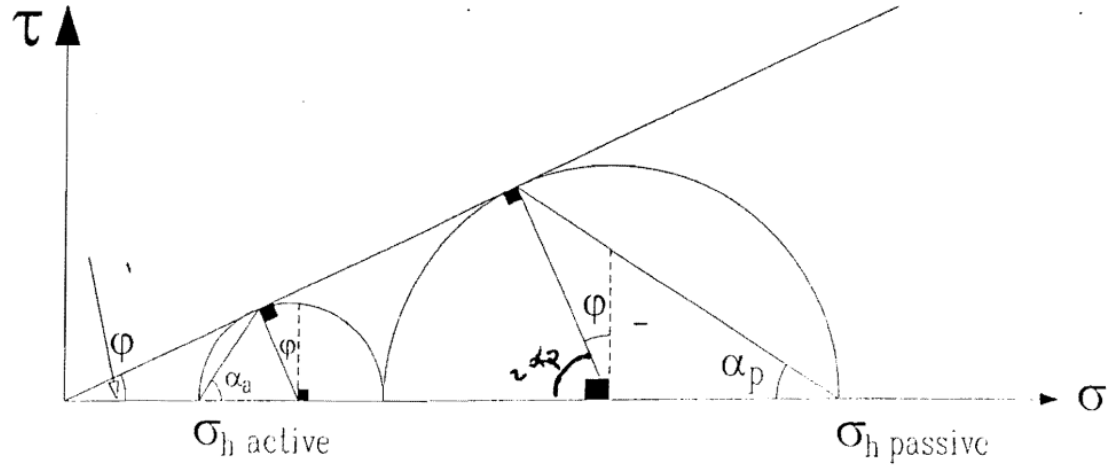
$$\sigma_1 (1 - \sin \phi) = \sigma_3 (1 + \sin \phi)$$

$$\frac{\sigma_3}{\sigma_1} = k_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

accordingly  $k_p = \frac{\sigma_1}{\sigma_3} = \frac{1 + \sin \phi}{1 - \sin \phi}$



# Slip Lines



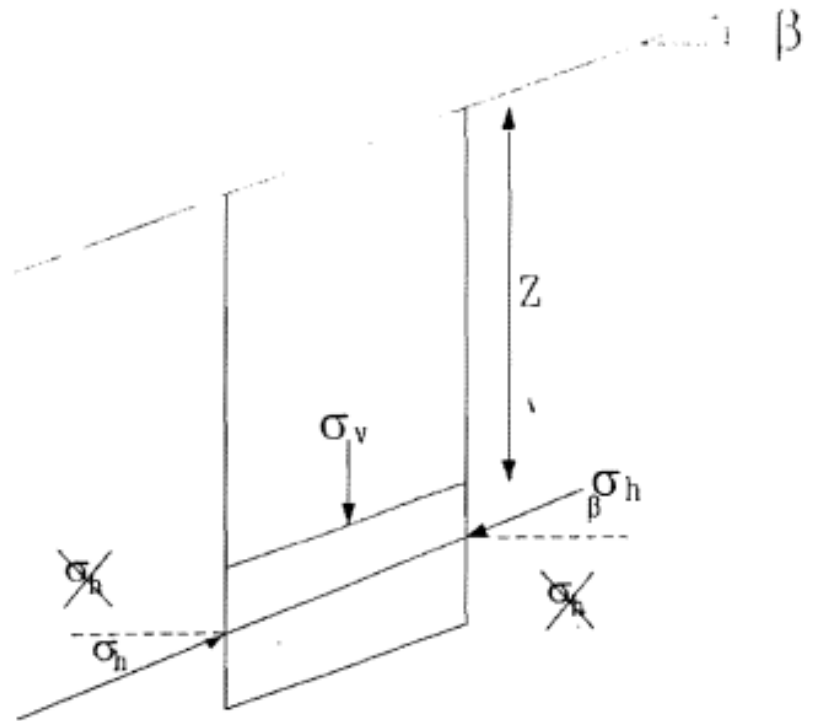
(a)

# Sloping Ground Surface

(i) Sloping G - S

From plastic equilibrium:-

$$K_a = \cos\beta \frac{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\Phi)}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\Phi)}}$$

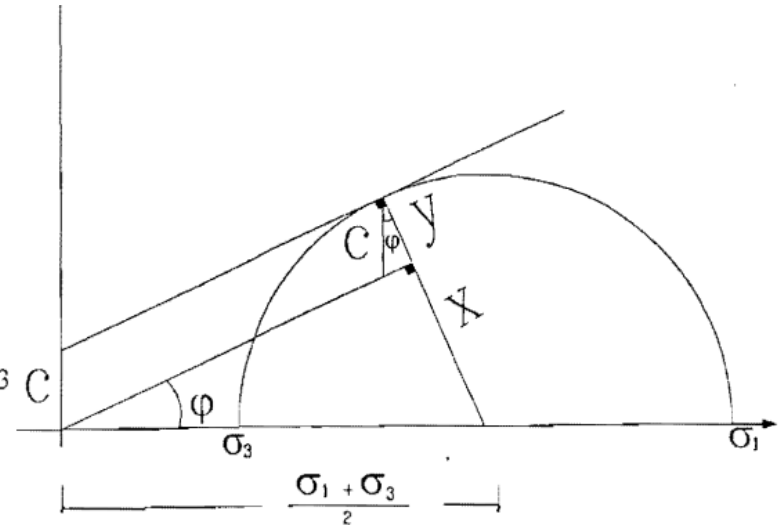


# Effect of Cohesion

$$x + y = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sin \phi \frac{\sigma_1 + \sigma_3}{2} + C \cos \phi = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sin \Phi (\sigma_1 + \sigma_3) + 2C \cos \Phi = \sigma_1 - \sigma_3$$



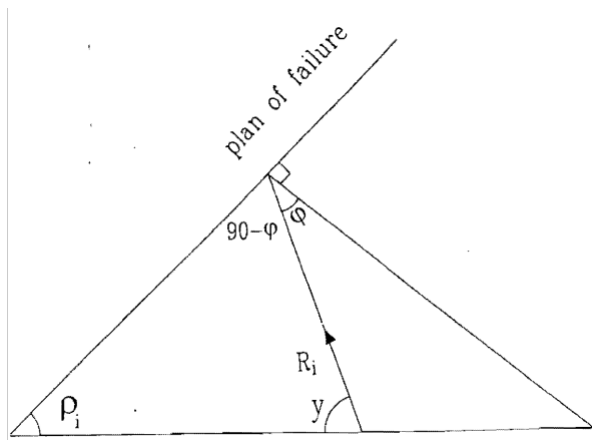
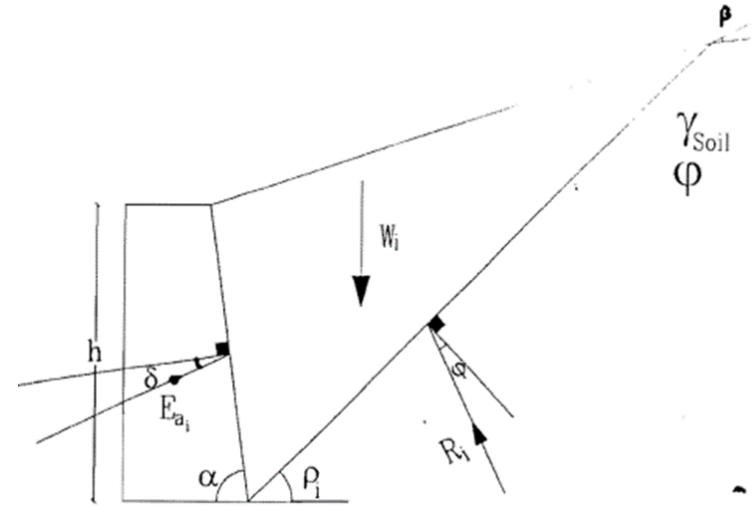
$$\sigma_3 (1 + \sin \Phi) = \sigma_1 (1 - \sin \Phi) - 2C \cos \Phi$$

$$\sigma_3 = \sigma_1 \frac{1 - \sin \phi}{1 + \sin \phi} - 2C \sqrt{\frac{(1 - \sin^2 \phi)}{(1 + \sin \phi)^2}}$$

$$\sigma_3 = \sigma_1 K_a - 2C \sqrt{K_a}$$

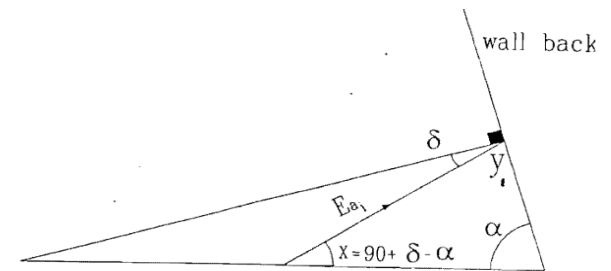
# Application of Limit Equilibrium

- $C = 0$
- Wall friction =  $\delta$
- Plane failure Surface
- Plane G.S



$$Y = 180 - 90 + \Phi - \rho_i$$

$$Y = 90 + \Phi - \rho_i$$



$$X + Y_i + \alpha = 180$$

$$Y_i + \delta = 90$$

$$X + \alpha + 90 - \delta = 180$$

$$X = 90 + \delta - \alpha$$

# Force Polygon

Force      Magn      Direct

$W_i$       ✓      ✓  
 $E_i$       ?      ✓  
 $R_i$       ?      ✓

Can be solved

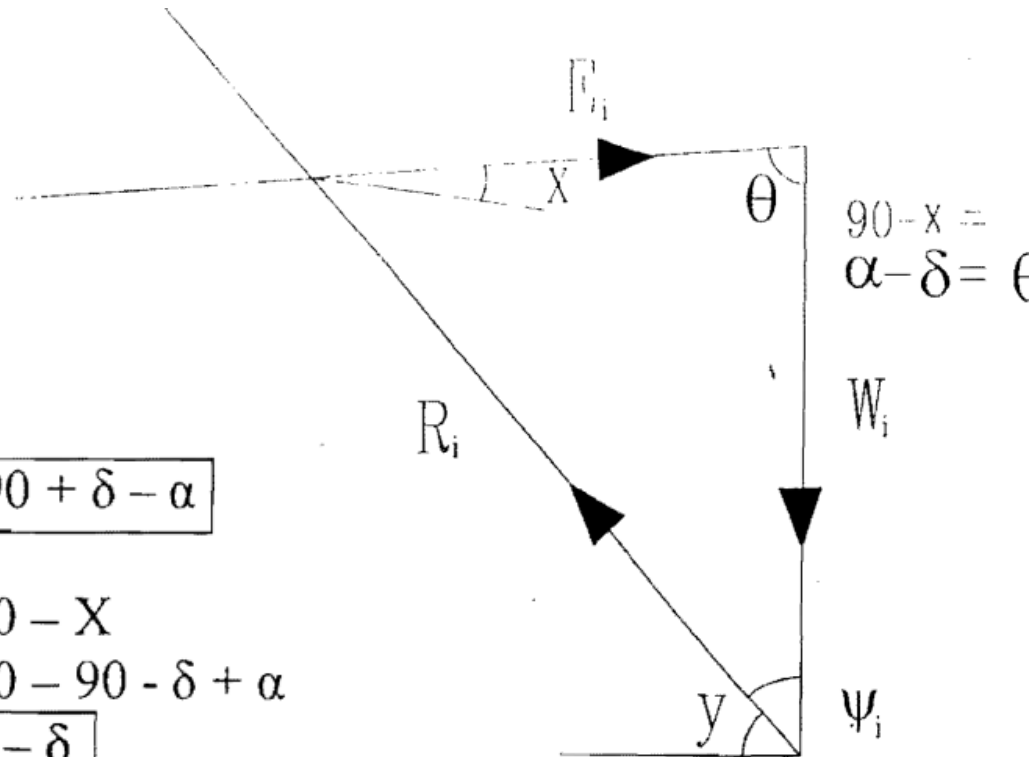
$$X = 90 + \delta - \alpha$$

$$\begin{aligned} \theta &= 90 - X \\ &= 90 - 90 - \delta + \alpha \end{aligned}$$

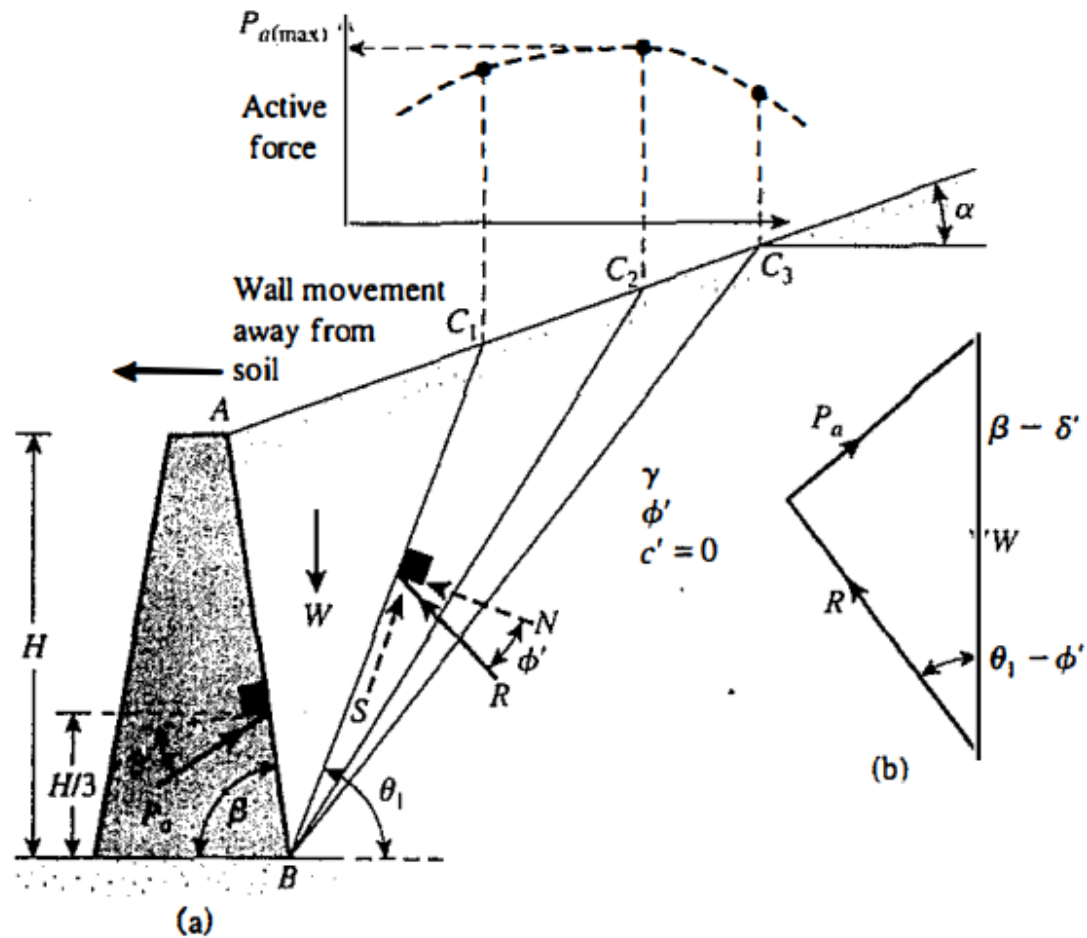
$$\theta = \alpha - \delta$$

$$\begin{aligned} \psi &= 90 - Y \\ &= 90 - (90 + \Phi - \rho_i) \end{aligned}$$

$$\psi_i = \rho_i - \Phi$$



# Coulomb's Solution for Active Case



# Closed Form Solution

$$\frac{E_i}{\sin \Psi_i} = \frac{W_i}{\sin (180 - (\theta + \Psi_i))}$$

$$E_i = \frac{\sin \Psi_i * W_i}{\sin (180 - (\theta + \Psi_i))}$$

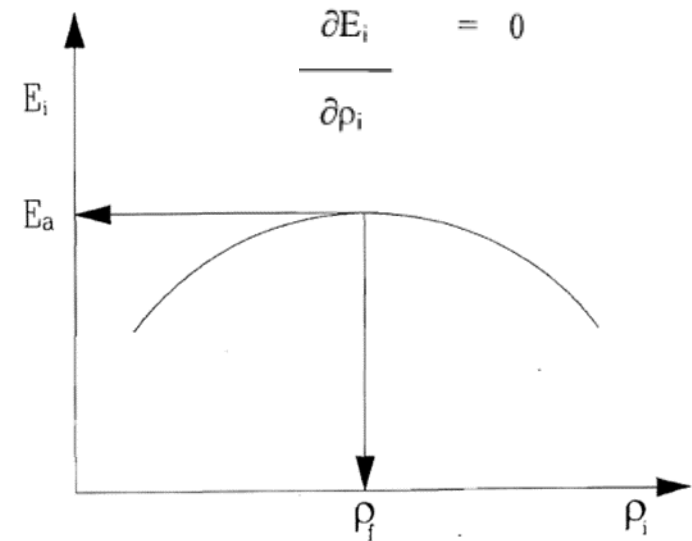
$$E_i = \frac{f(\gamma_{soil}, \beta, \rho_i, \alpha, \phi, h)}{\sin (180 - (\theta + \rho_i - \phi))}$$

$$E_i = \frac{f(\gamma_{soil}, \beta, \rho_i, \alpha, \phi, h)}{\sin (180 - (\alpha - \delta + \rho_i - \phi))} = f(const, \rho_i)$$

# Active EP Coefficient

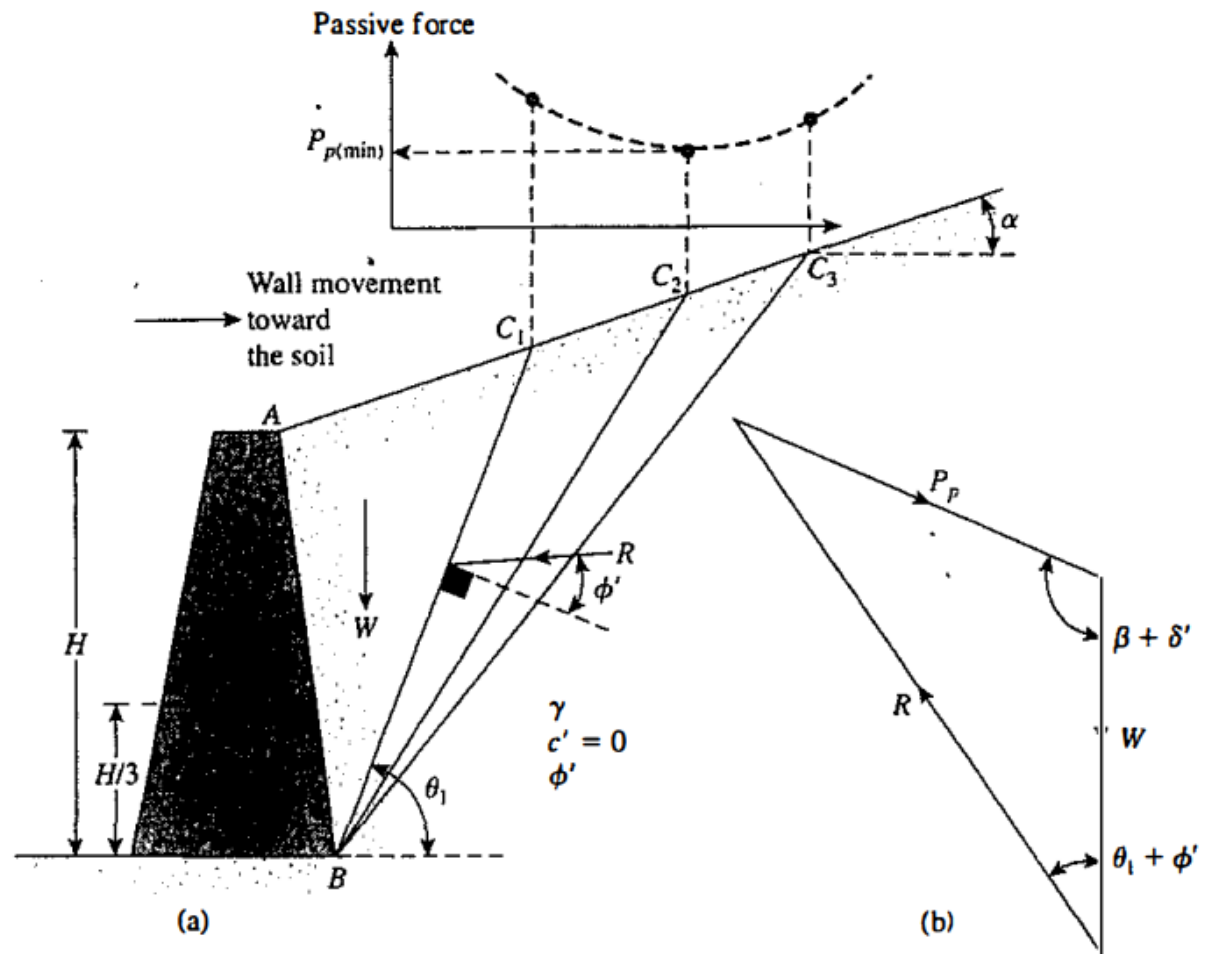
$$E_a = f(\text{cons}, \rho_f) = \frac{1}{2} \gamma_{\text{soil}} * h^2 * k_a$$

$$k_a = \frac{2E_a}{\gamma_{\text{soil}} h^2}$$

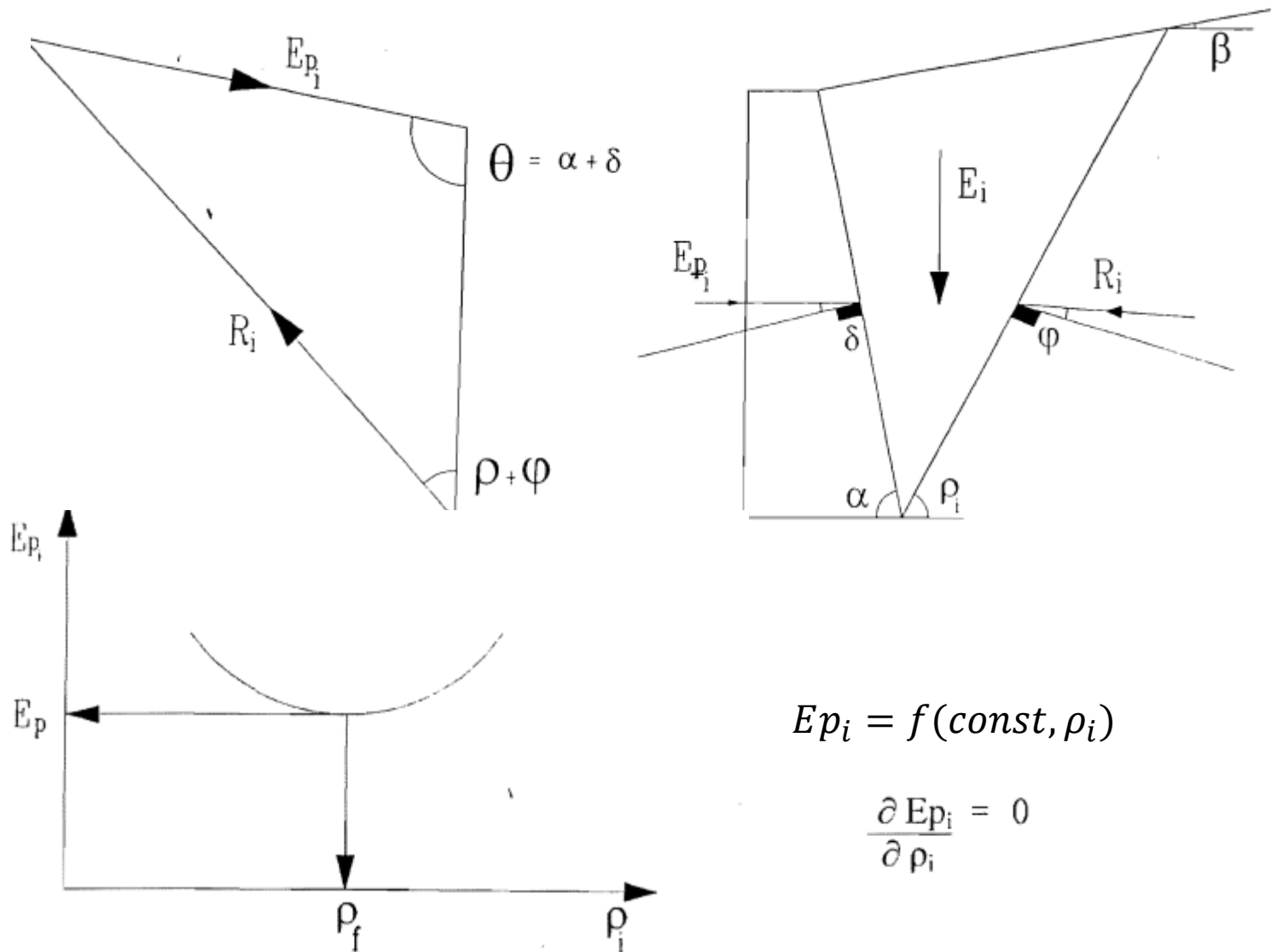


$$k_a = \frac{\text{Sin}^2(\alpha + \phi)}{\text{Sin}^2 \alpha \text{Sin}(\alpha - \delta) \left[ 1 + \sqrt{\frac{\text{Sin}(\phi + \delta) \text{Sin}(\phi - \beta)}{\text{Sin}(\alpha - \delta) \text{Sin}(\alpha + \beta)}} \right]^2}$$

# Coulomb Solution for Passive Case



# Closed form Solution is Possible



# Passive EP Coefficient

$$E_p = f(const, \rho_f)$$

$$E_p = f(const, \rho_f) = \frac{1}{2} \gamma_{soil} * h^2 * k_p$$

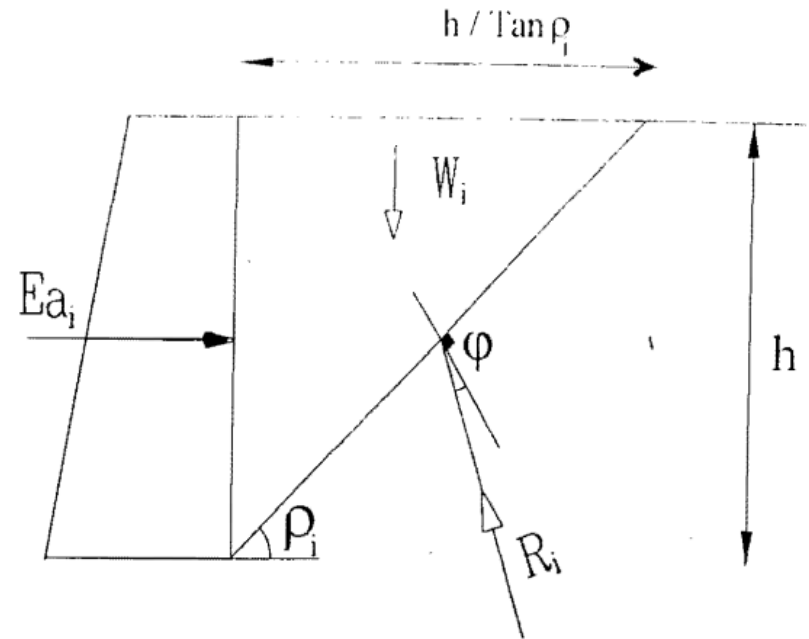
$$k_p = \frac{\sin^2(\alpha - \phi)}{\sin^2 \alpha \sin(\alpha + \delta) \left[ 1 - \sqrt{\frac{\sin(\phi + \delta) \sin(\phi + \beta)}{\sin(\alpha + \delta) \sin(\alpha + \beta)}} \right]^2}$$

# Applying Rankine Assumptions

$$\beta = 0$$

$$\alpha = 90^\circ$$

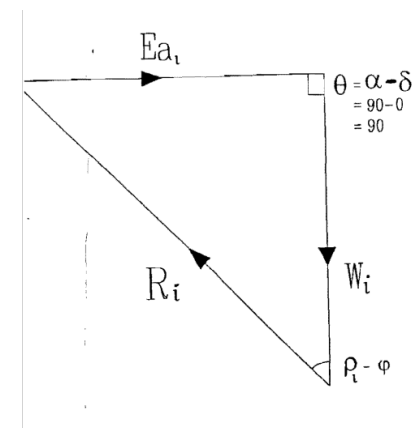
$$\delta = 0$$



$$\tan(\rho_i - \phi) = \frac{E_{ai}}{W_i}$$

$$E_{ai} = w_i \tan(\rho_i - \phi)$$

$$w_i = 1/2 h \frac{h\gamma}{\tan \rho_i} = \frac{h^2\gamma}{2 \tan \rho_i}$$



# Similar Results

$$E_a \text{ max, } \frac{\partial E_{ai}}{\partial \rho_i} = 0$$

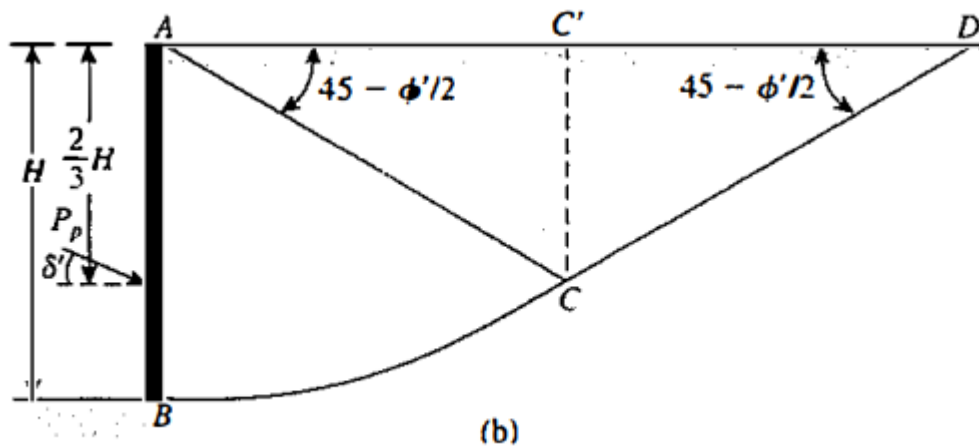
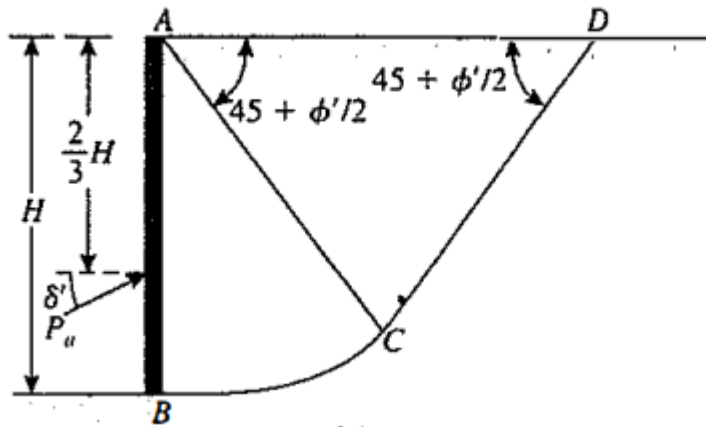
$$\rho_f = 45 + \frac{\phi}{2} \quad E_a = \frac{\gamma h^2}{2} \tan^2 (45 - \phi/2)$$

$$K_a = \tan^2 (45 - \phi/2) = \frac{1 - \sin \phi}{1 + \sin \phi}$$

**Similarly**

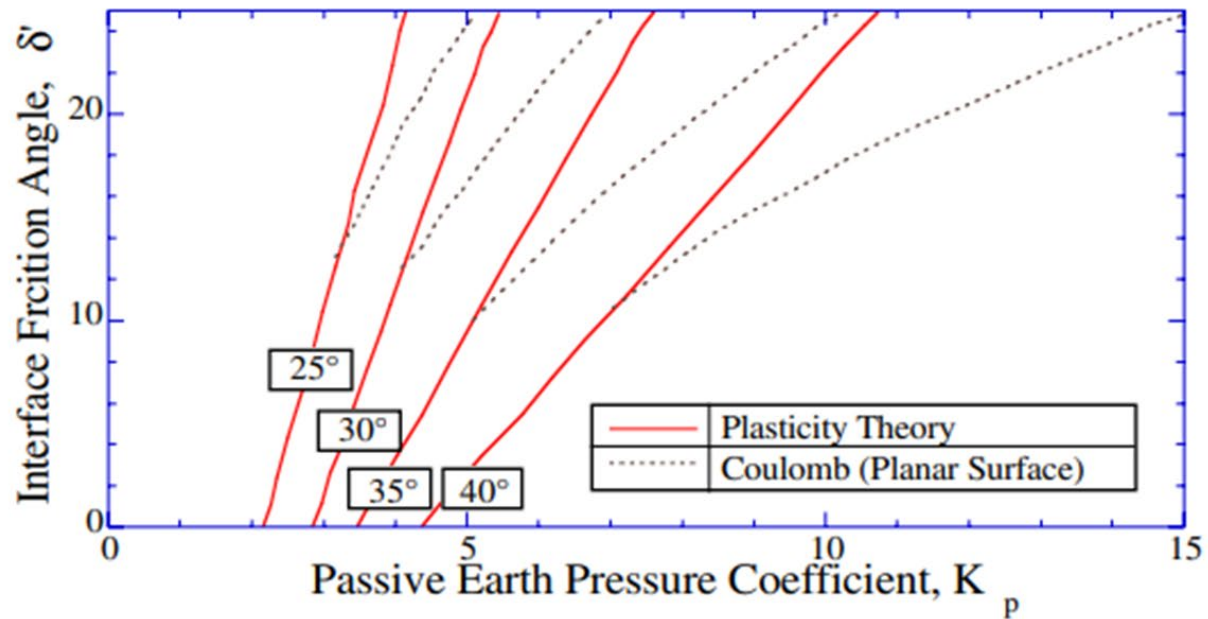
$$K_p = \tan^2 (45 + \phi/2) = \frac{1 + \sin \phi}{1 - \sin \phi}$$

# Issues with Coulomb Solution

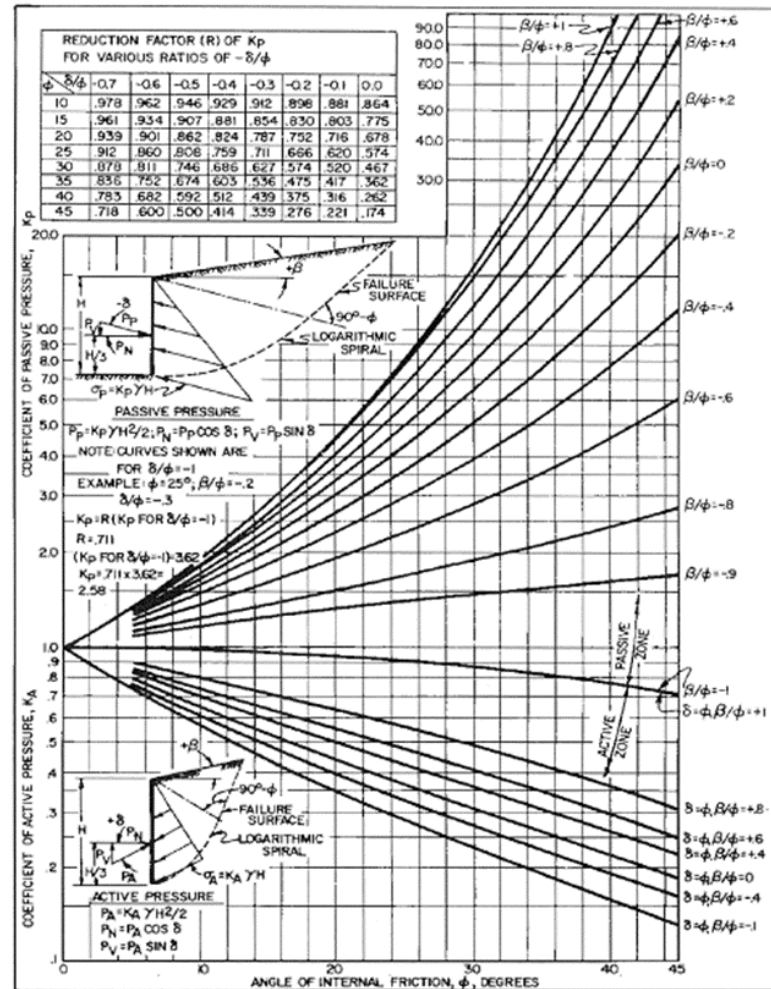


# Errors in the passive Case

- Coulomb analyses with a planar surface can greatly overestimate  $K_p$  values (see below). In practice, values of  $K_p$  can be calculated following procedures outlined in DM7. It is usually conservative to use the Rankine  $K_p$  value (corresponding to  $\delta' = 0$ ).



# Modified Passive Coefficient





**Thank you**