PBW 620 Advanced Soil Mechanics
PBW 584 Applied Soil Mechanics

Public Works Department

MSc. Degree

Spring Semester

Prof. Dr. Sherif A. Akl
Soil Mechanics and Foundations Research Lab
Faculty of Engineering- Cairo University

Lecture Two

ANALYTICAL AND NUMERICAL APPROACHES FOR THE BEARING CAPACITY PROBLEM

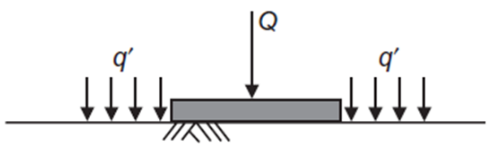
Lecture Outline

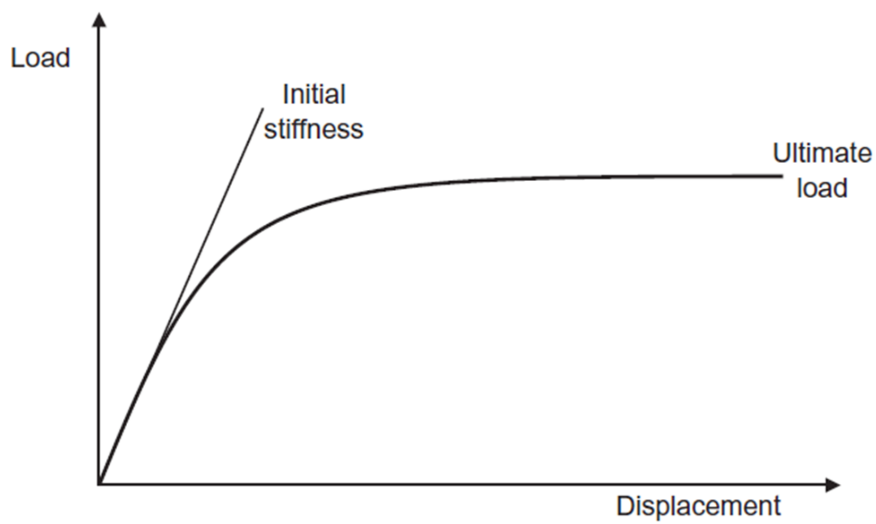
- Limit Analysis of Bearing Capacity
- Method of Slip lines
- Application of Numerical Analysis
- Advantages of Numerical Analysis

The bearing Capacity Problem

A simple surface foundation problem

$$q_{\max} = \frac{Q_{\max}}{A} = i_c d_c s_c N_c^{\text{strip}} c' + i_q d_q s_q N_q^{\text{strip}} q' + i_\gamma d_\gamma s_\gamma N_\gamma^{\text{strip}} B\gamma'$$





<u>Conventional Analysis</u>

1. Settlement Analysis using elastic soil properties and Boussinesq equations

2. Ultimate load using bearing capacity equations.

<u>Assumptions</u>

- Dry or saturated soil
- Layers are not considered
- Mohr Coulomb failure criterion only

Let's try limit analysis

- A strip footing on soil surface
- c = 50 kPa, Ø=0, γ'=0

$$q_{max} = cN_c = 5.14c$$
$$q_{max} = 257 \text{ kPa}$$

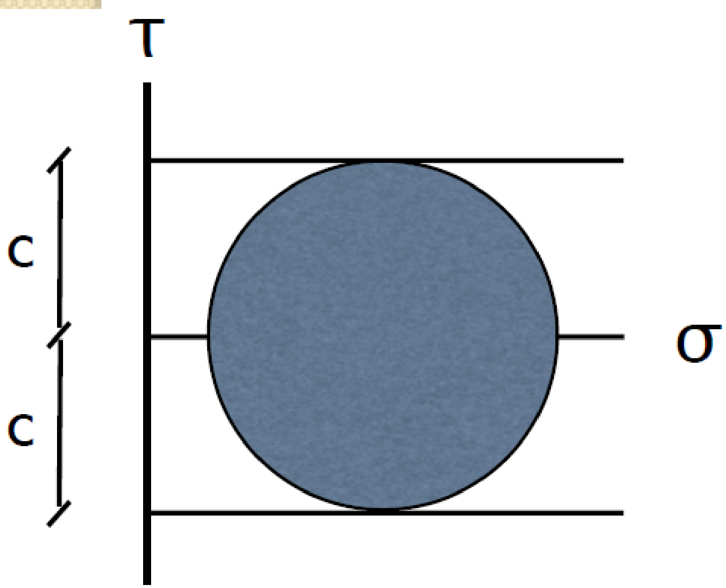
Lower Bound Theorem

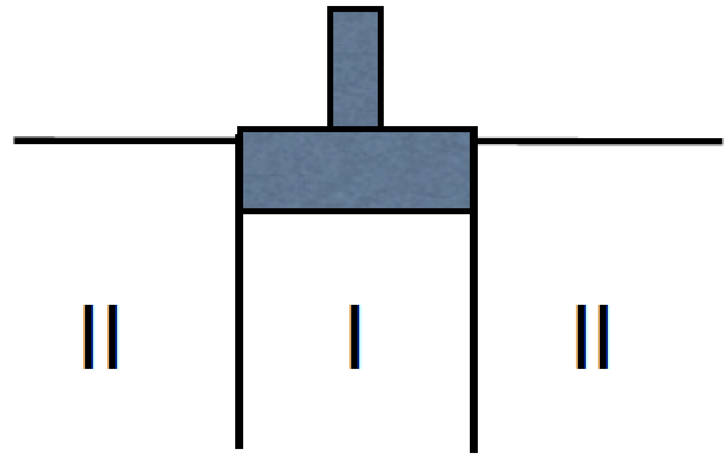
If a distribution of stresses can be found within the body which satisfies equilibrium at all points in the body and does not exceed yield criteria then exterior loads are lower bound loads to the true collapse loading conditions.

This is actually what we need: A Design value for the external loads that will always be less than the true collapse value

Distribution of Stresses
A Stress Field

Tresca Failure Criterion





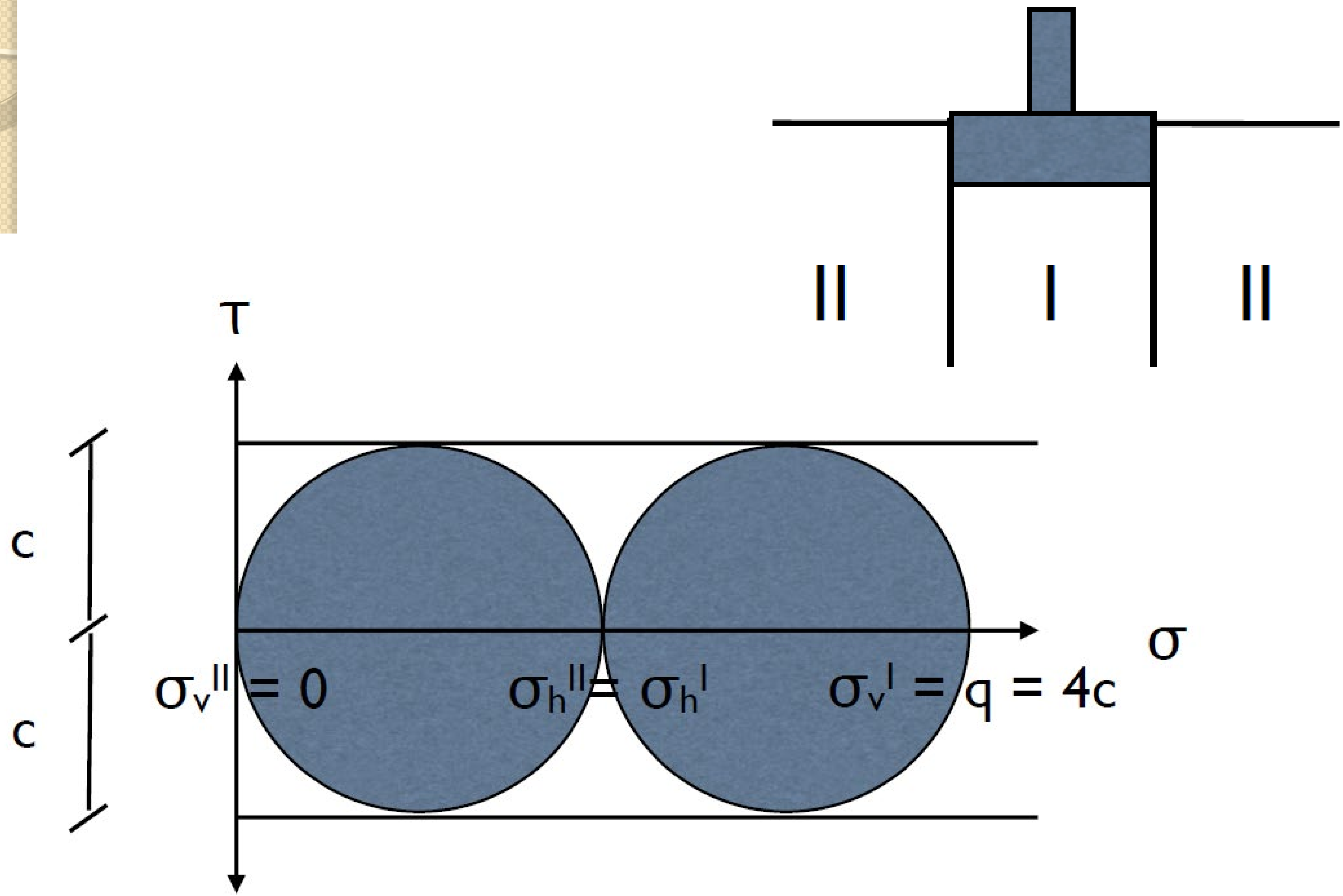
Lines of Discontinuity

$\sigma_v^{II} = 0$

$\sigma_h^{II} = \sigma_h^{I}$

$\sigma_v^{I} = q$

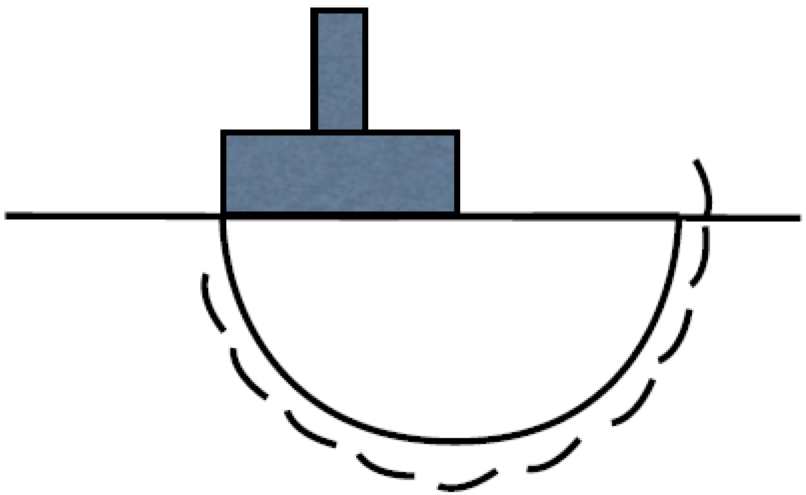
The Maximum Load (LB)

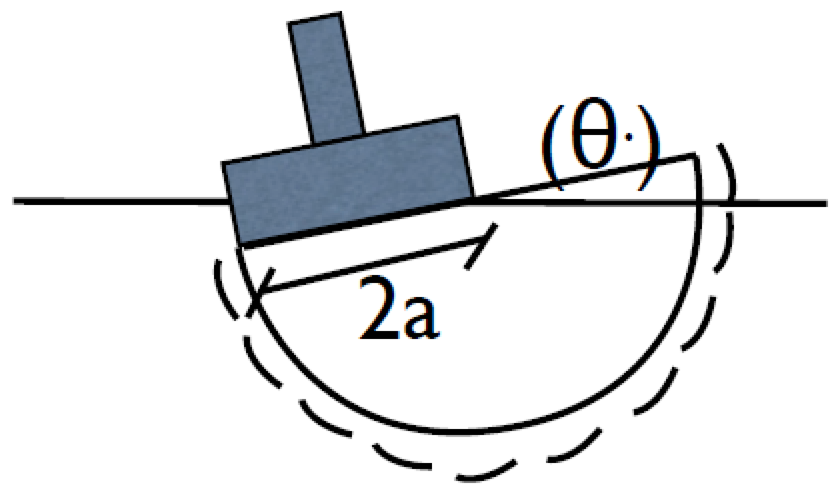


Upper Bound Theorem

If a distribution of velocity field (a mechanism of failure) can be found such that the rate of work done by the externally applied loads is larger than the energy dissipated internally by the material then exterior loads are upper bound loads to the true collapse loading conditions.

Mechanism of Failure





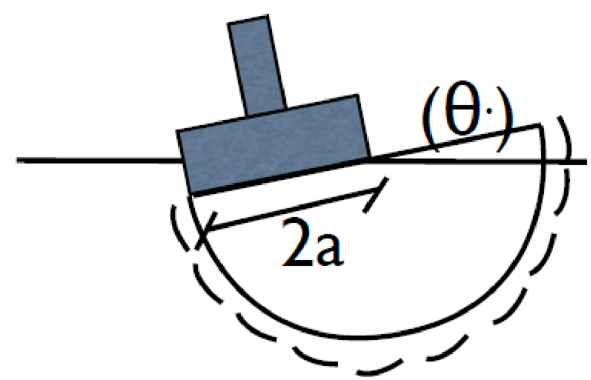
The Minimum Load

$$\dot{W}^{e} = \int_{2a}^{0} q(\dot{\theta})x\,dx = 2qa^2\dot{\theta}$$

$$\dot{W}^{i} = \pi 2a * c * 2a\dot{\theta}$$

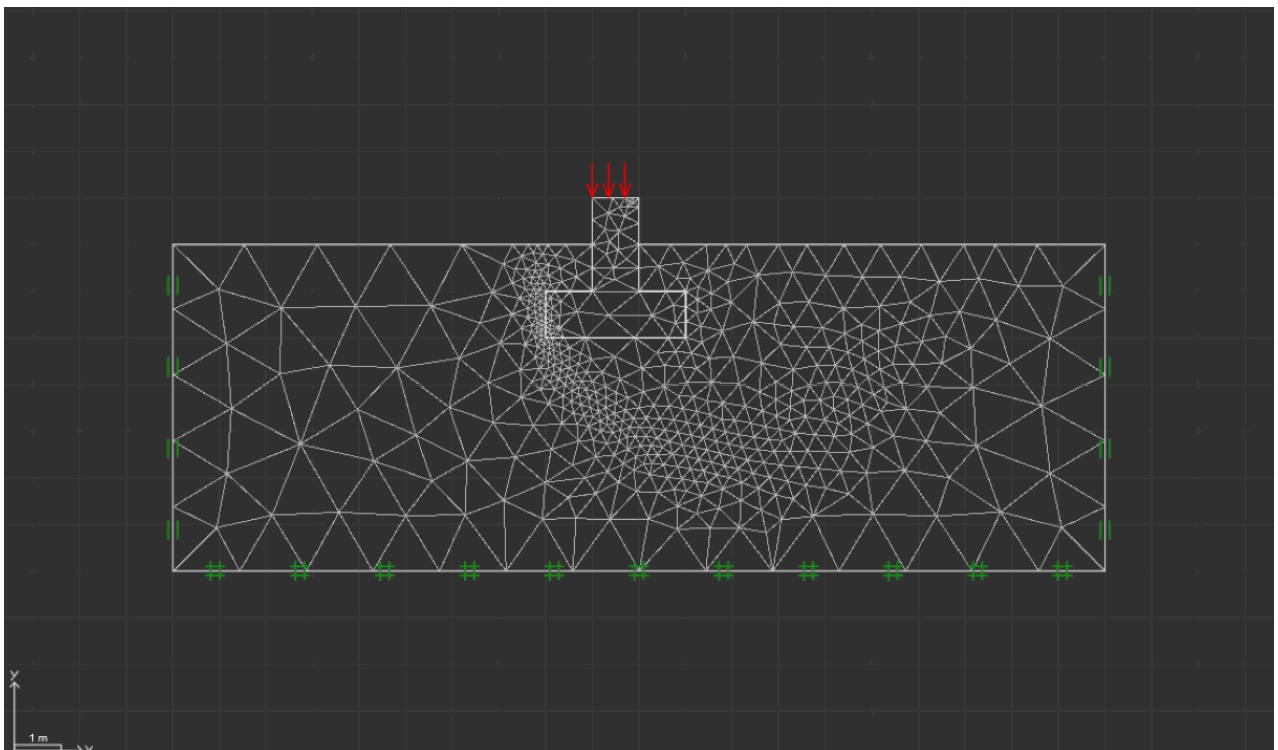
$$q = 2\pi c = 6.28c$$

$$4c < q < 6.28c$$



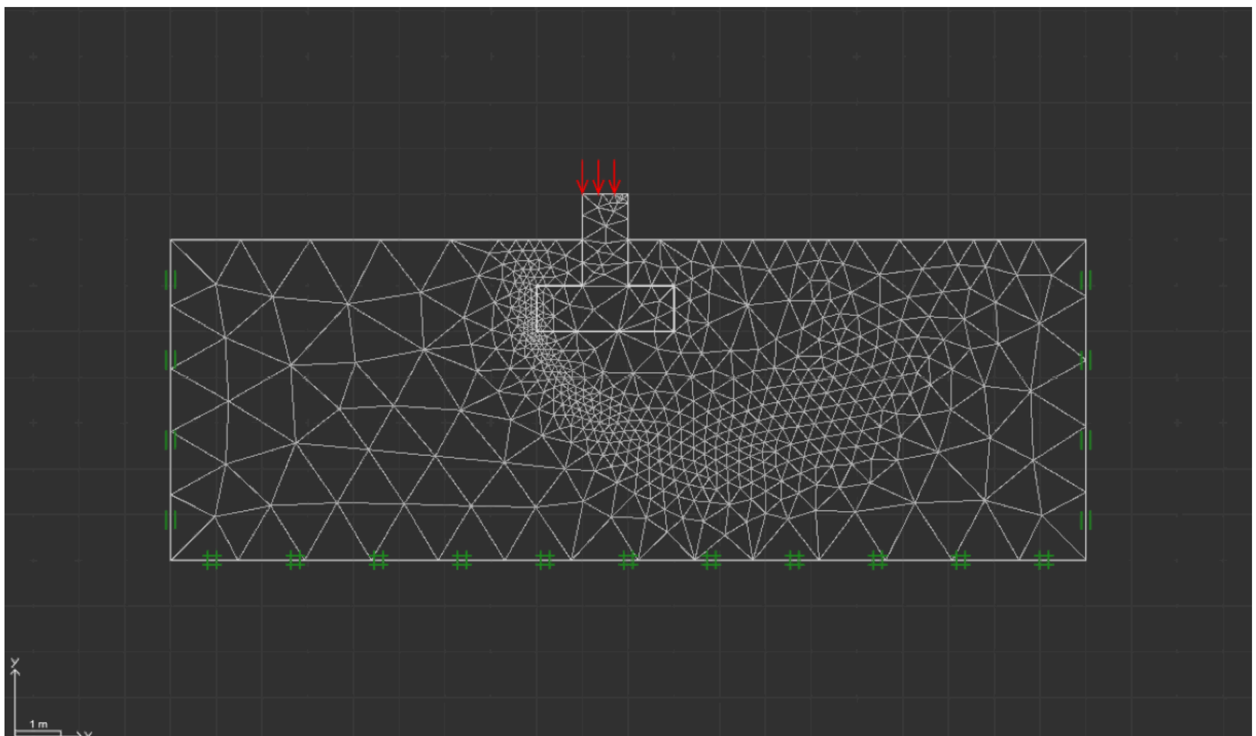
Lower Bound Solution

- The optimized stress field associated with the maximum loads
- Solution does not observe compatibility



Upper Bound Solution

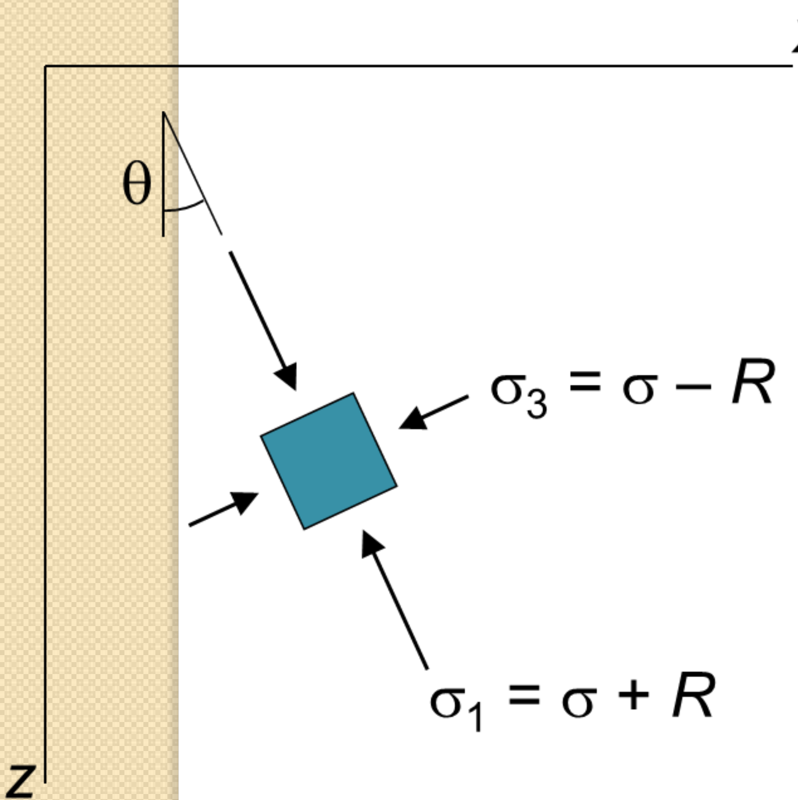
- The optimized failure mechanism associated with the minimum loads
- Solution does not observe equilibrium

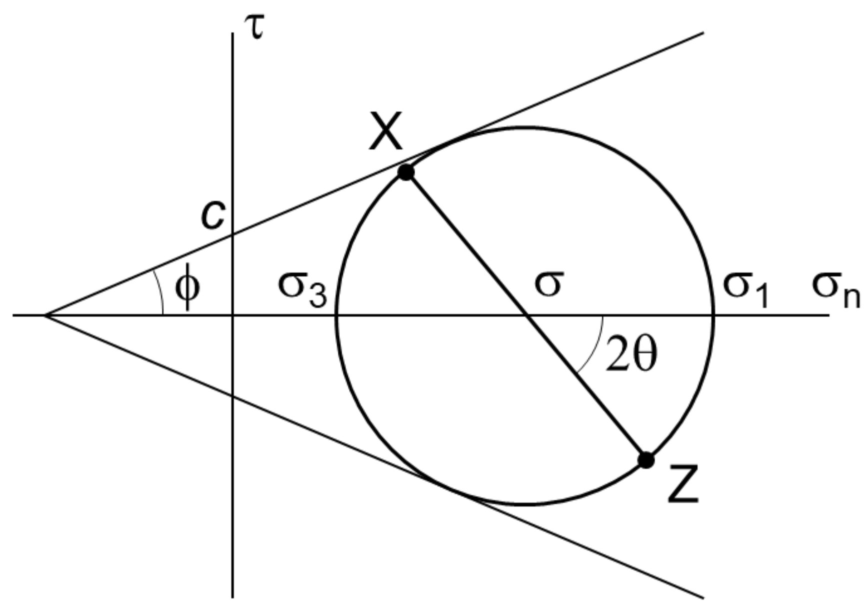


Method of Slip Lines

To define a 2D stress field, e.g. in *x*-*z* plane

- normally need 3 variables (σ_{xx}, σ_{zz}, τ_{xz})
- if assume soil is at yield, only need 2 variables (σ, θ)



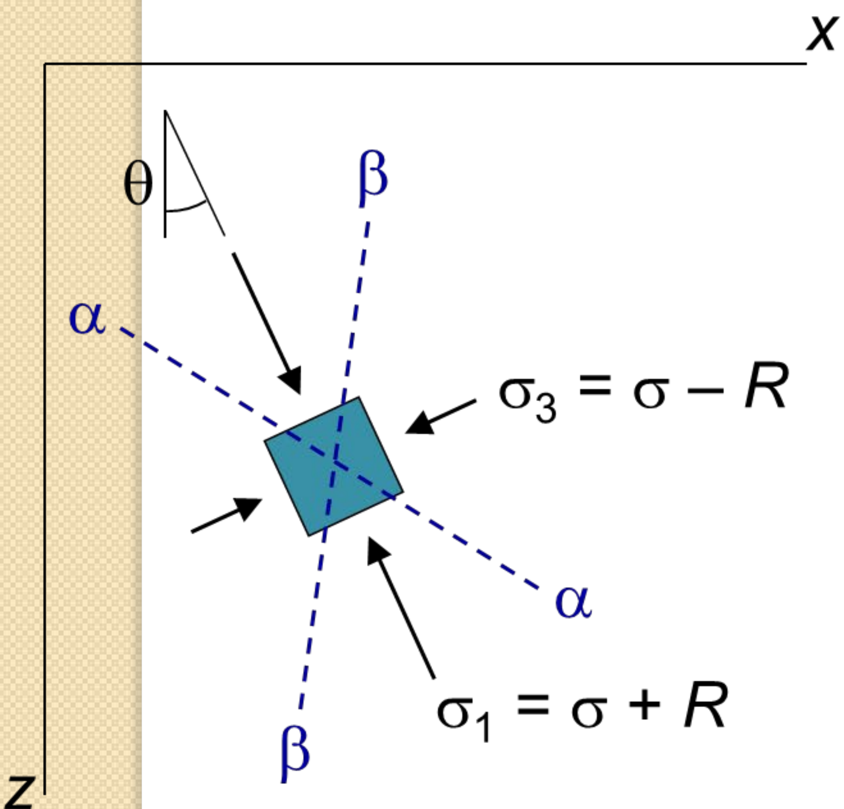


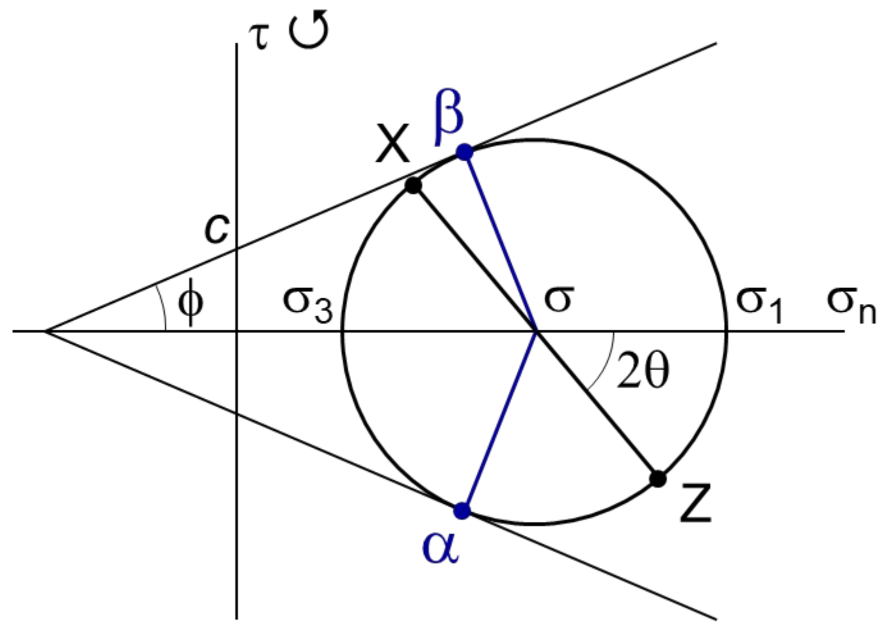
$R = c\cos\phi + \sigma\sin\phi$ M-C

[$R = f(\sigma,\theta)$ general]

Method of Slip Lines

- The Slip line geometry at every point can be determined from the pole of the Mohr circle





$$R = c\cos\phi + \sigma\sin\phi \qquad \text{M-C}$$

$$[\, R = f(\sigma,\theta) \text{ general} \,]$$

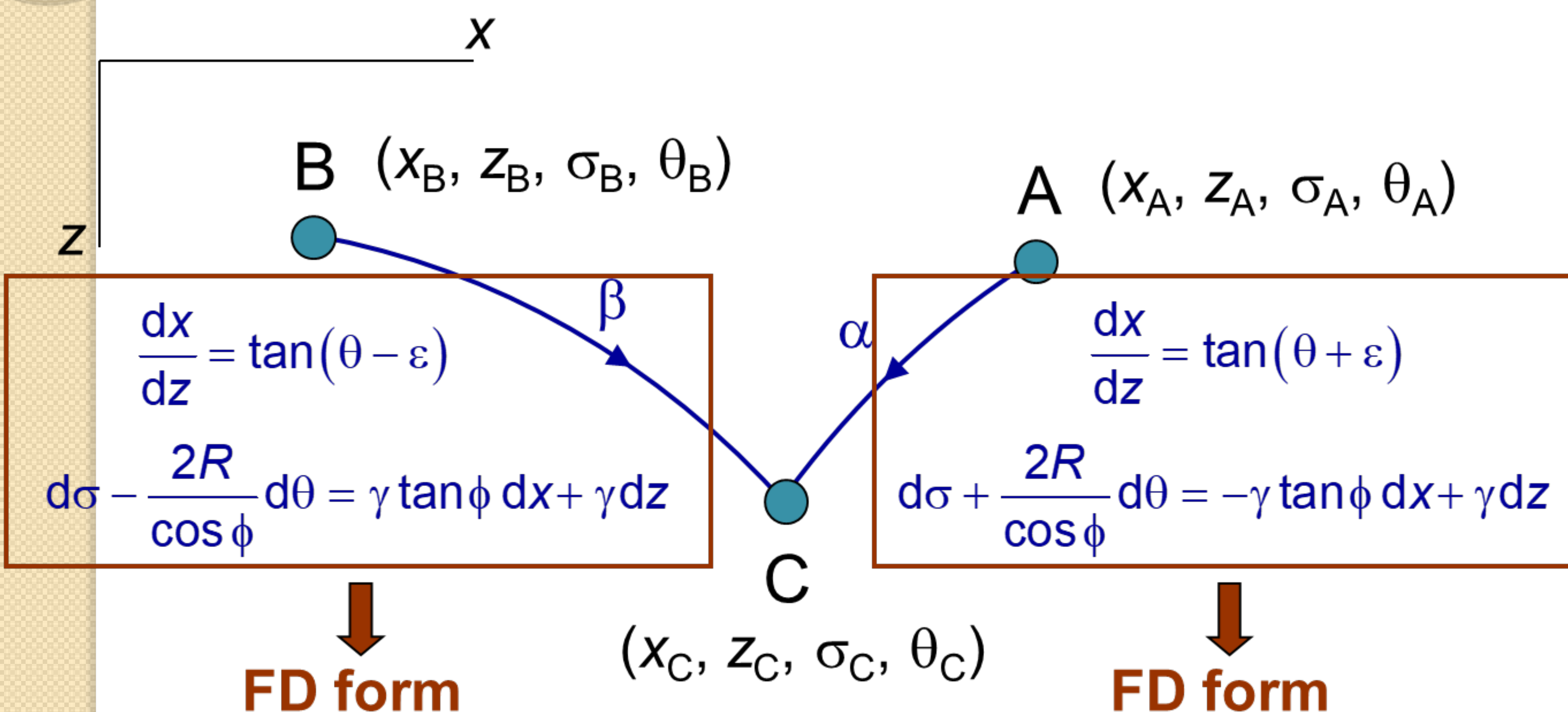
Lower bound stress field

- Substitute stresses-at-yield (in terms of σ, θ) into equilibrium equations

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \qquad \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = \gamma$$

- Result is a pair of hyperbolic PDEs in σ, θ
- Characteristic directions turn out to coincide with α and β ‘slip lines’ aligned at $\theta \pm \varepsilon$
- Use α and β directions as curvilinear coords $\rightarrow$ obtain a pair of ODEs in σ, θ (easier to integrate)
- Solution can be marched out from known BCs

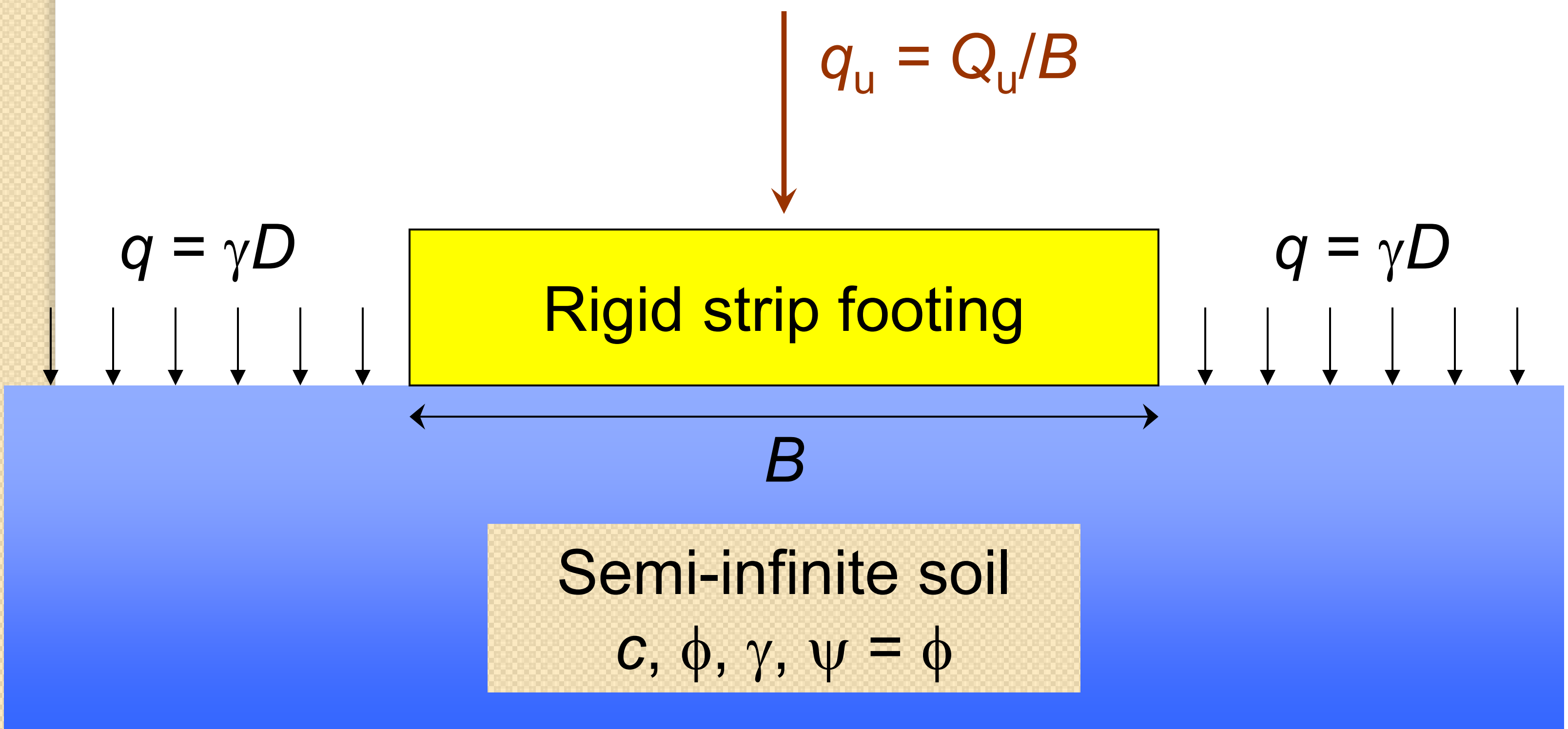
Lower bound stress field



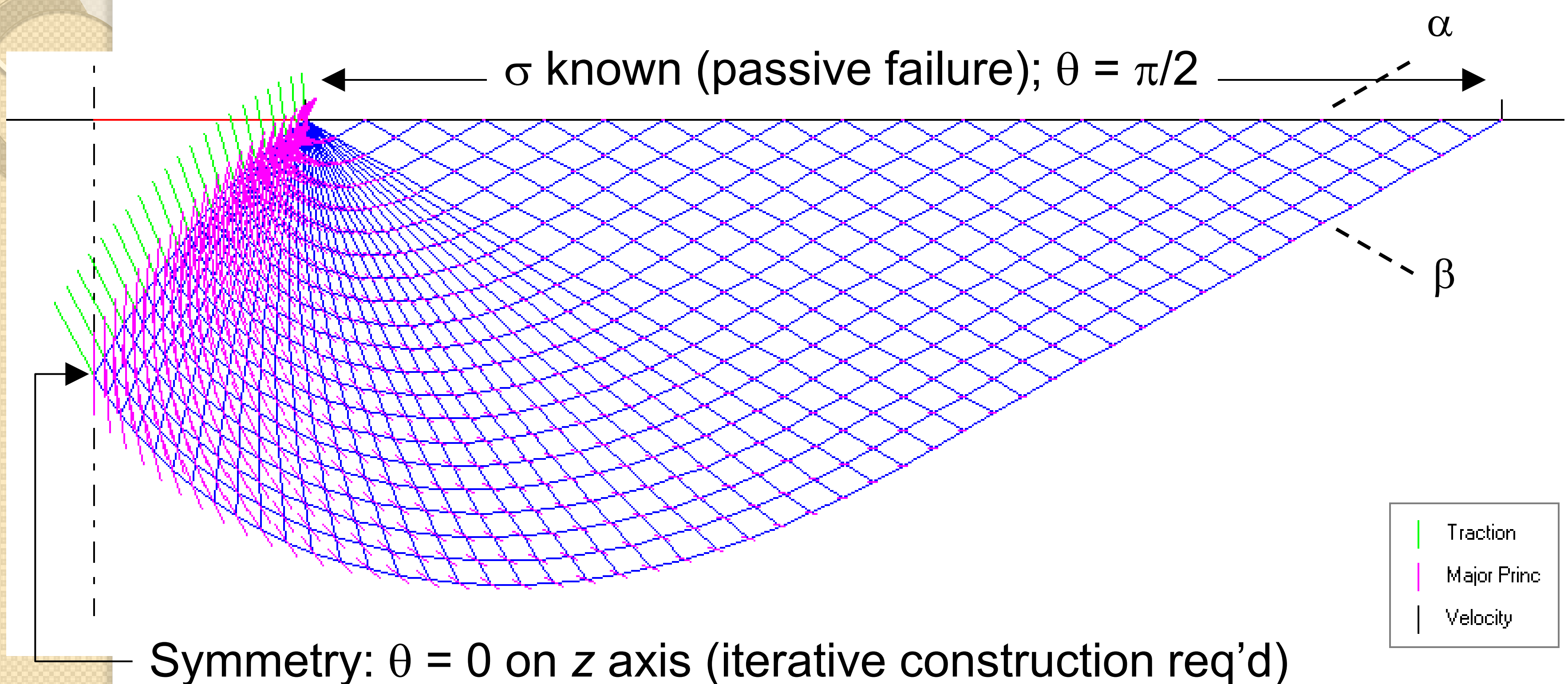
Bearing capacity – ABC software

- Idealised problem (basis of design methods):

Central, purely vertical loading



Example problem: stress field (partial)



- Shape of 'false head' region emerges naturally
- q_u from integration of tractions
- Solution not strict LB until stress field extended:

Potts 2003 footing on weightless soil

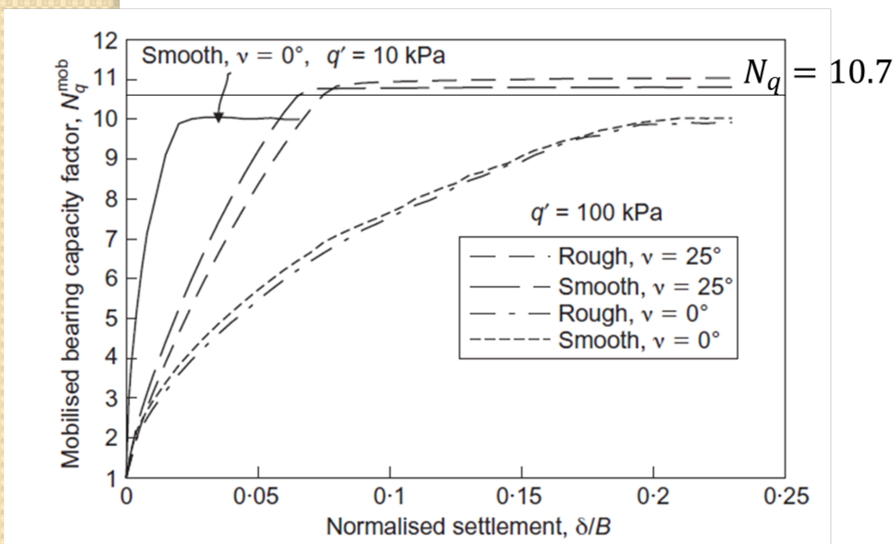
$$\phi = 25^o, c' = 0, \gamma = 0$$

$$N_q^{mob} = \frac{Q}{Aq'}$$

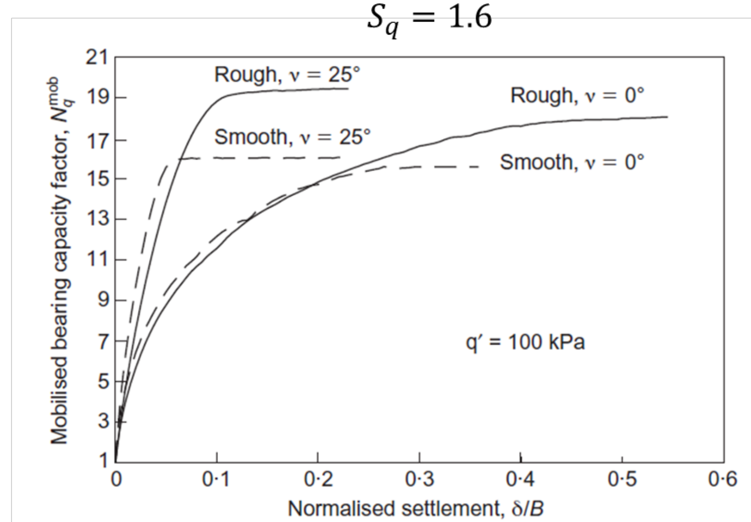
$$q_{max} = \frac{Q_{max}}{A} = s_q N_q^{strip} q' + s_\gamma N_\gamma^{strip} B\gamma'$$

$Typically\ 1.2 < S_q < 1.5$

$S_q = 1.6$



Strip Footing



Circular Footing

Potts 2003 – footing with no surcharge

$$\phi = 30^o, c' = 0, q' = 100, \gamma = 1.8t/m^3$$

$$N_\gamma^{mob} = \frac{Q}{AB\gamma'}$$

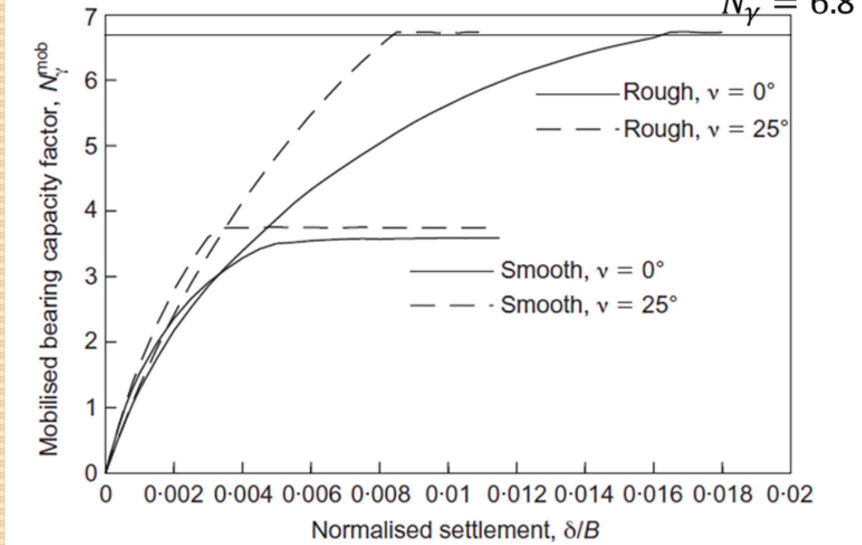
$$q_{\max} = \frac{Q_{\max}}{A} = s_q N_q^{\text{strip}} q' + s_\gamma N_\gamma^{\text{strip}} B\gamma'$$

$Typically\ 0.6 < S_\gamma < 0.9$

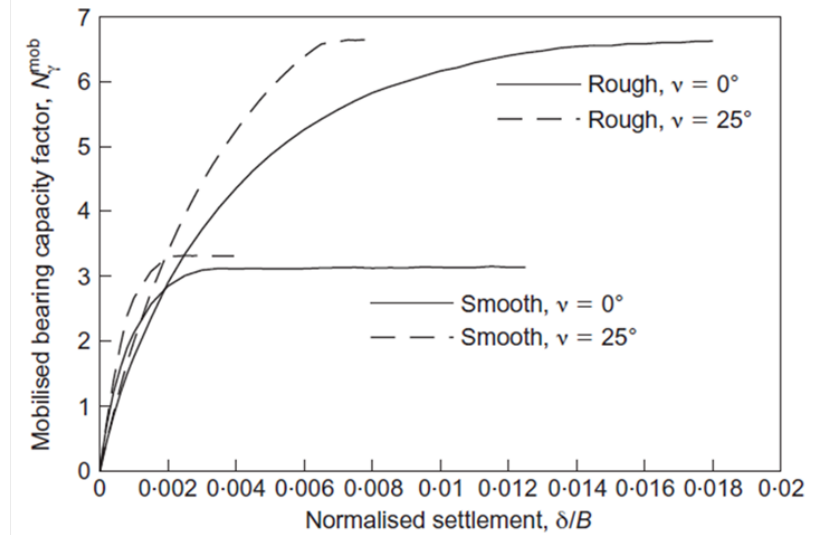
Hansen or Meyerhof

$N_\gamma = 6.8$

$S_\gamma = 0.9{\sim}1.0$



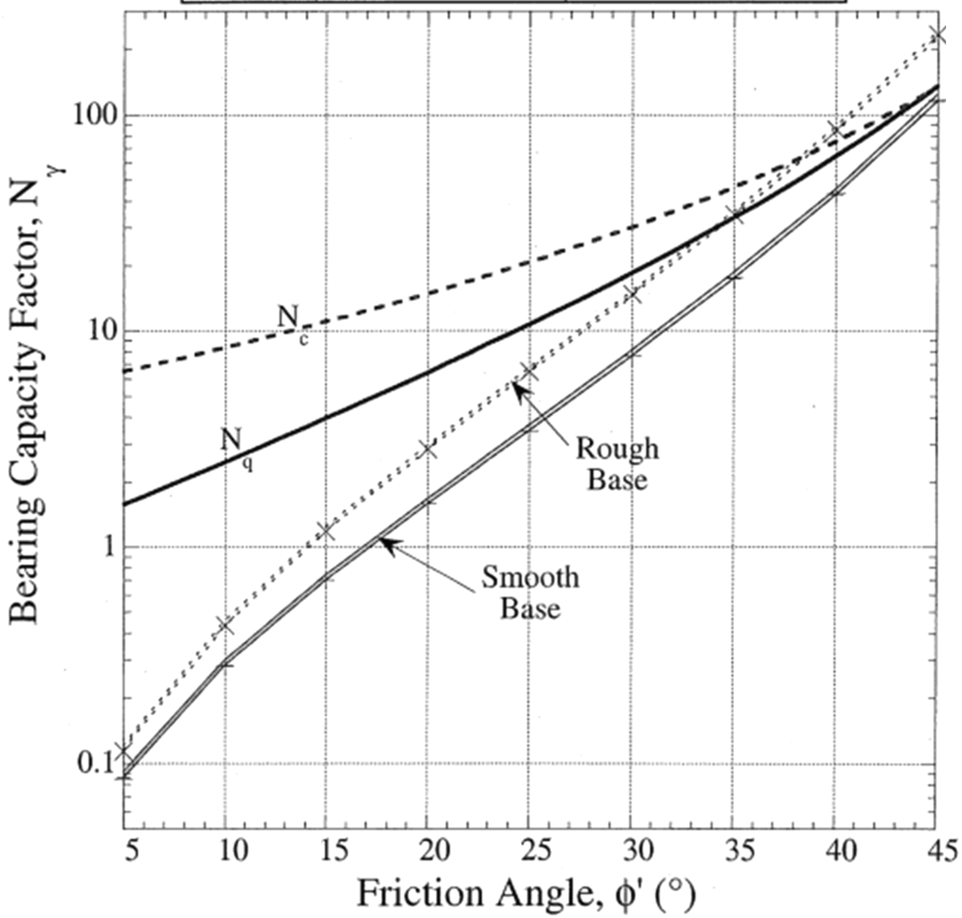
Strip Footing

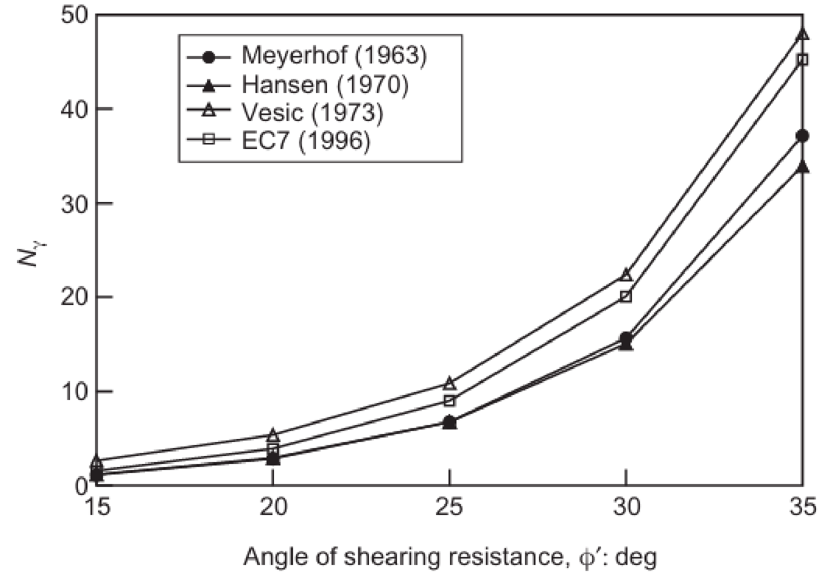


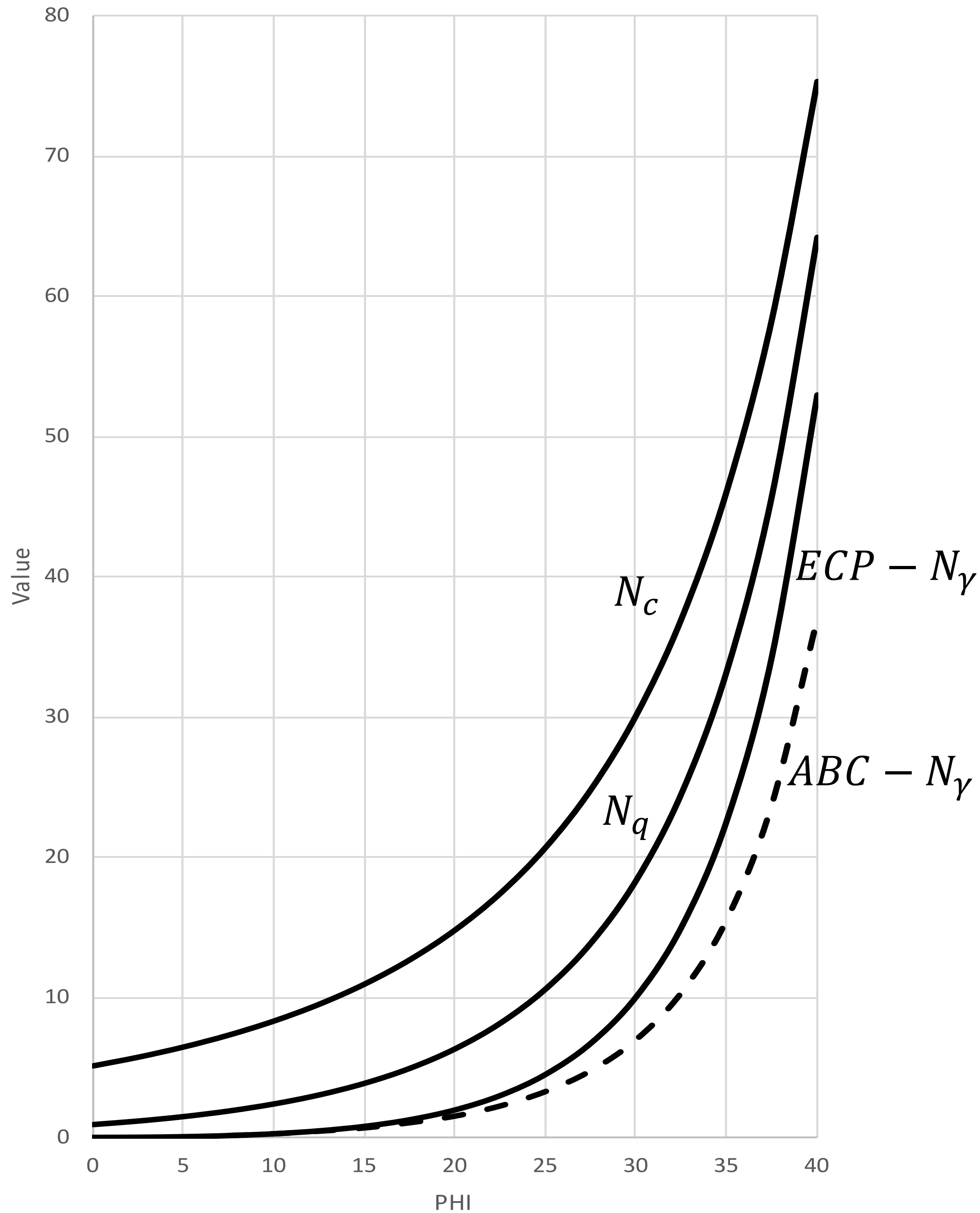
Circular Footing

Nγ

N_γ	Method of Stress Characteristics (Martin, 2004)	Numerical Limit Analyses (Hijaj et al., 2004)
Smooth	+	———
Rough	×	········



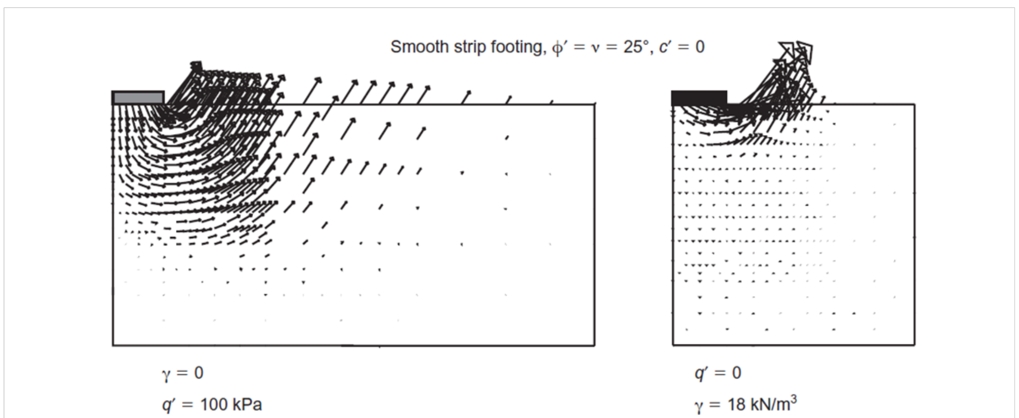




Value
PHI
0
10
20
30
40
50
60
70
80
0
5
10
15
20
25
30
35
40
N_c
N_q
$ECP - N_\gamma$
$ABC - N_\gamma$

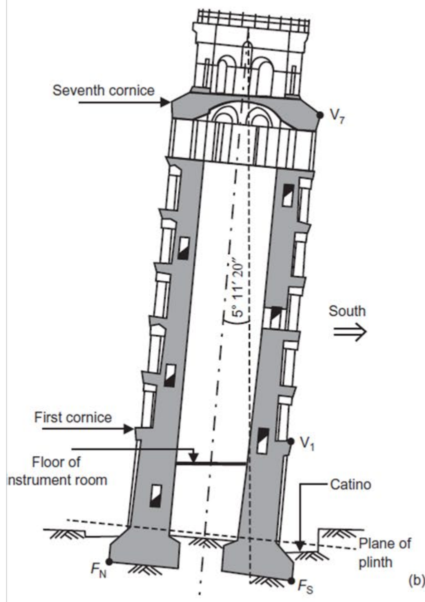
Mechanisms

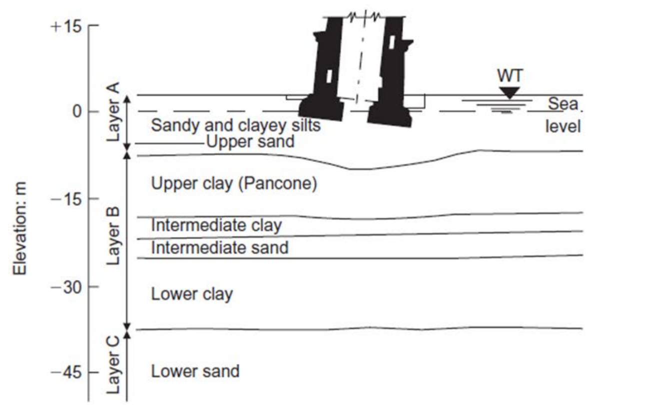
- Complete load displacement curve
- Effect of soil dilation angle
- Failure mechanism



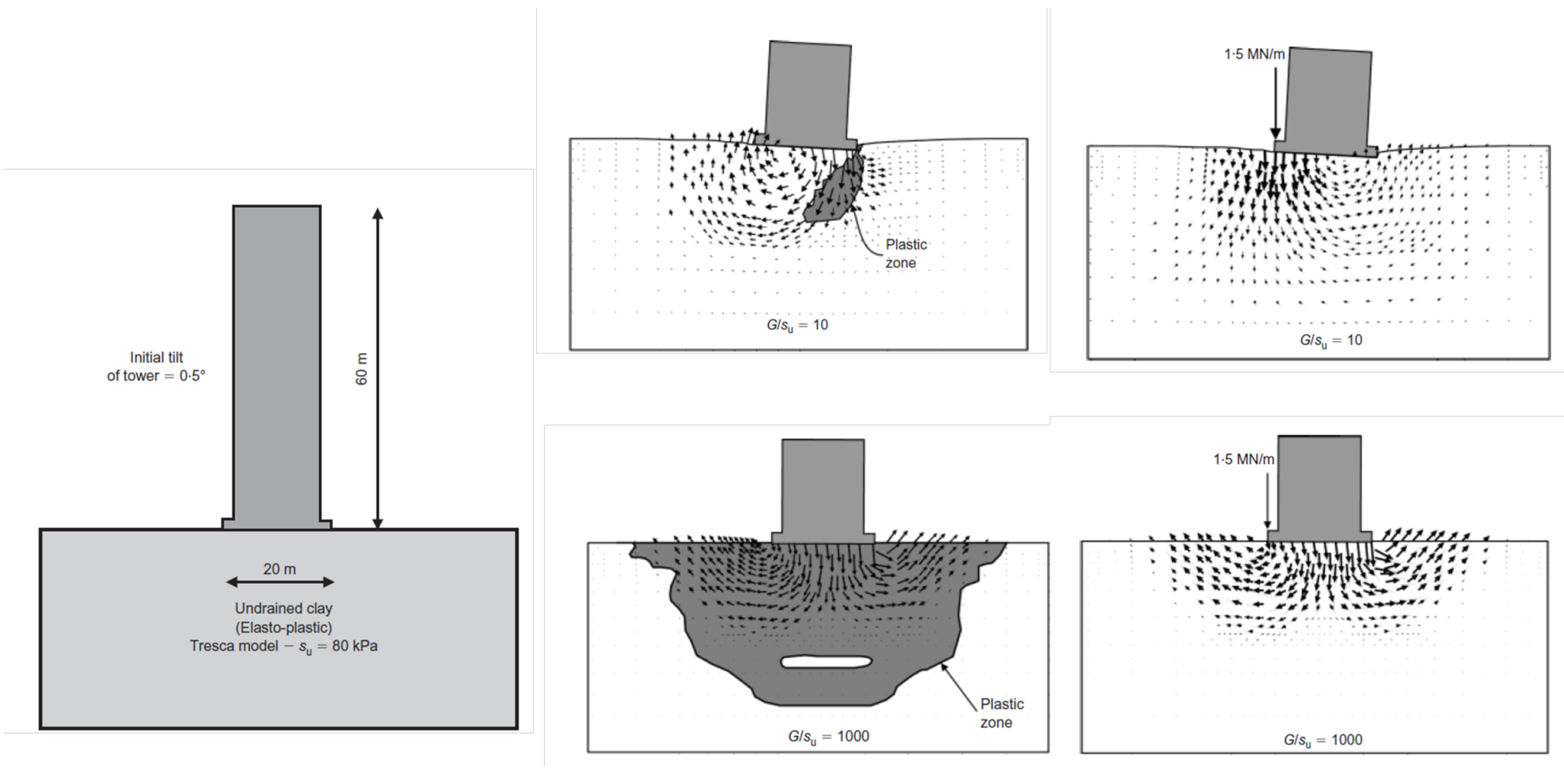
- Different mechanisms for the two problems which are otherwise superimposed in conventional analysis.
- Typically bearing capacity equation is conservative for cases where q’ and gamma have values.

Pisa Tower


Seventh cornice
V7
5° 11′ 20″
South
First cornice
V1
Floor of
instrument room
Catino
Plane of
plinth
FN
FS
(b)


+15
0
−15
−30
−45
Elevation: m
Layer A
Layer B
Layer C
WT
Sea
level
Sandy and clayey silts
Upper sand
Upper clay (Pancone)
Intermediate clay
Intermediate sand
Lower clay
Lower sand

Leaning Instability



Thank you