PBW 620 Advanced Soil Mechanics II
PBW 584 Applied Soil Mechanics

Public Works Department

MSc. Degree

Spring Semester

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Faculty of Engineering- Cairo University

Part II

- APPLICATION OF NUMERICAL METHODS IN BEARING CAPACITY PROBLEM
- EARTH PRESSURE

Acknowledgment

- Finite Element Modeling in Geotechnical Engineering (Potts and Zdravkovic)
- Dr. C. M. Martin University of Oxford
- Prof. Andrew Whittle lecture notes, Massachusetts Institute of Technology
- Prof. Dr. Hani Lotfi

Outline of Part II

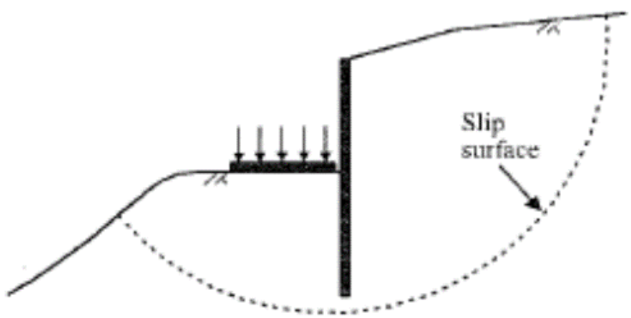
Lecture	Description
First	Comparison between numerical methods and conventional Methods of Analysis
Second	Application of Analytical and Numerical methods to the Bearing Capacity Problem
Third	Earth Pressure using conventional analysis
Fourth	Special Conditions affecting earth pressure in design - Part 1
Fifth	Special Conditions affecting earth pressure in design – Part 2

Lecture 1 Outline

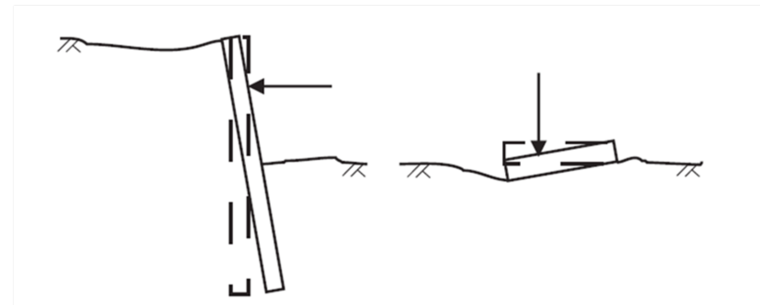
- Geotechnical Analysis Framework
- What are the conventional methods
- What are the numerical methods
- Comparisons and Discussion

Geotechnical Eng. Problem

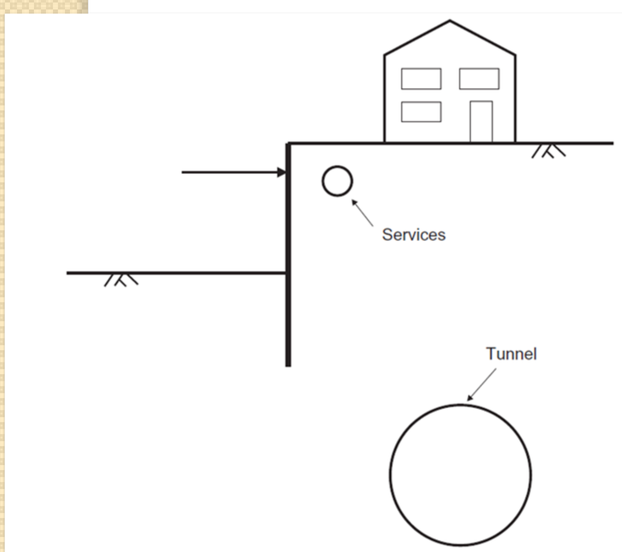
Stability



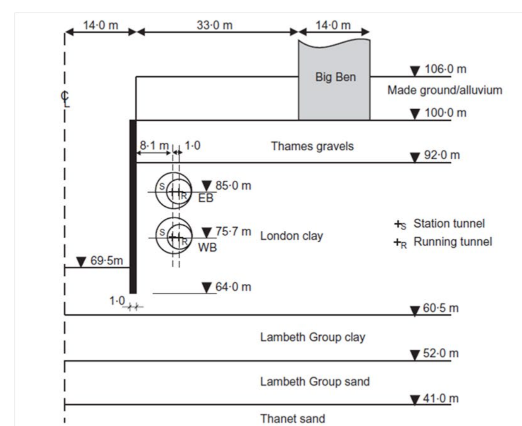
Deformations



Movement



Adjacent Structures



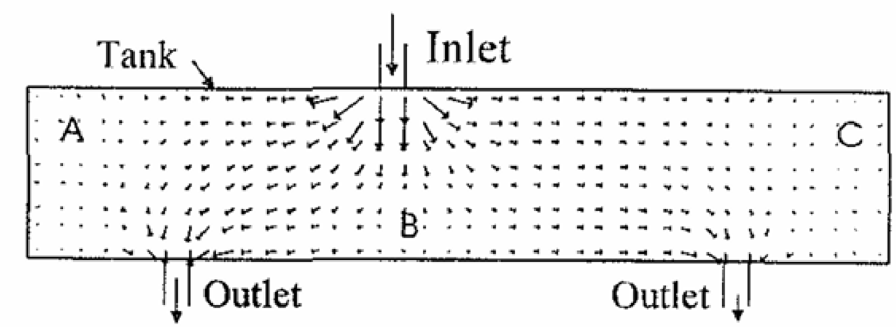
Analysis Framework

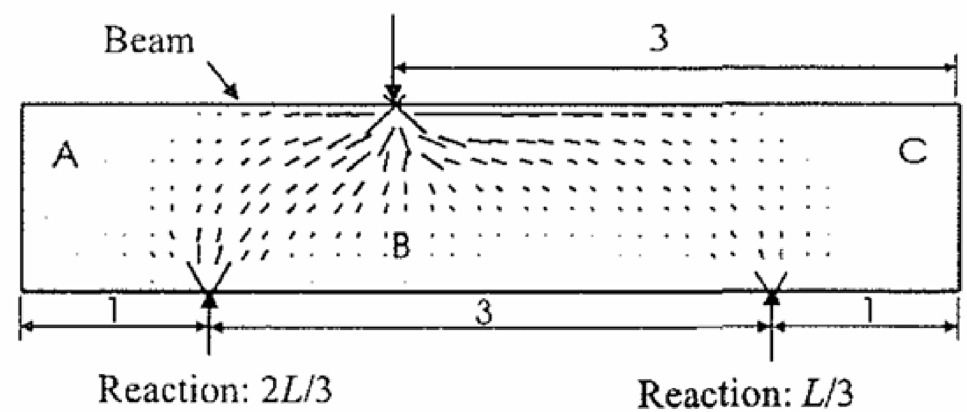
I- Equilibrium

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \gamma = 0$$

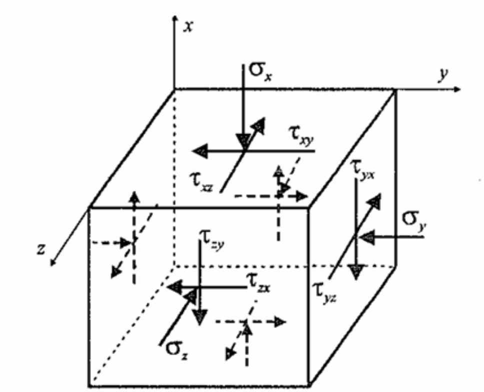
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$$

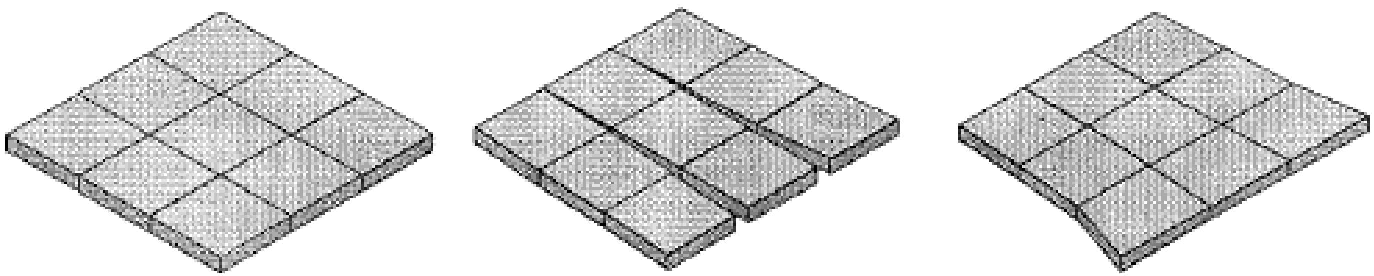




- Force Boundary Conditions
- Displacement Boundary Conditions



II-Compatibility



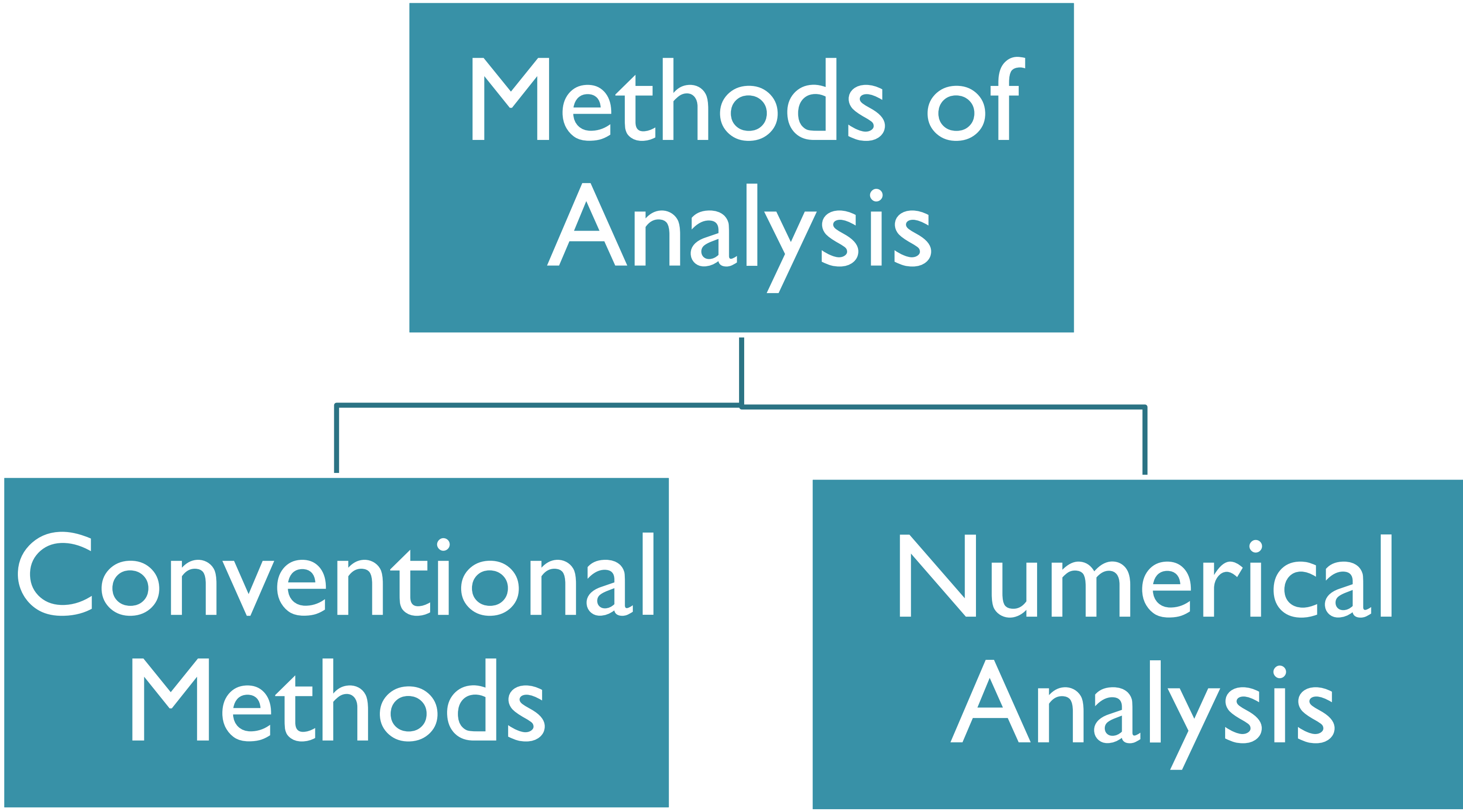
a) Original b) Non-compatible c) Compatible

$$\varepsilon_x = -\frac{\partial u}{\partial x} \; ; \; \varepsilon_y = -\frac{\partial v}{\partial y} \; ; \; \varepsilon_z = -\frac{\partial w}{\partial z}$$

$$\gamma_{xy} = -\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \; ; \; \gamma_{yz} = -\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \; ; \; \gamma_{xz} = -\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$$

III-Constitutive Models

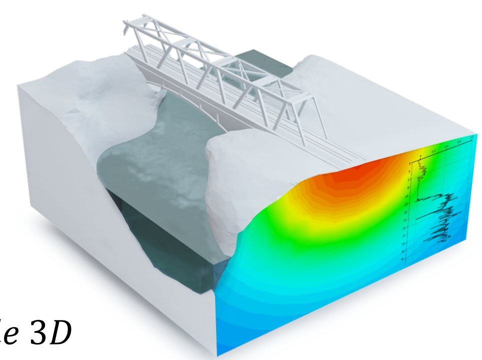
$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\tau_{xy} \\ \Delta\tau_{xz} \\ \Delta\tau_{zy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_z \\ \Delta\gamma_{xy} \\ \Delta\gamma_{xz} \\ \Delta\gamma_{zy} \end{Bmatrix}$$


Methods of Analysis
Conventional Methods
Numerical Analysis

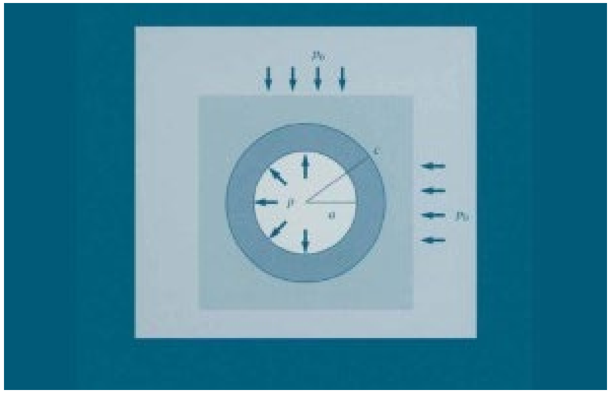
Conventional Methods

Closed Form

- The solution is exact in a theoretical sense but is still an approximation to the real problem.
- Soil behaves in an isotropic linear-elastic manner.
- Practical application is limited to cases with geometric symmetries.



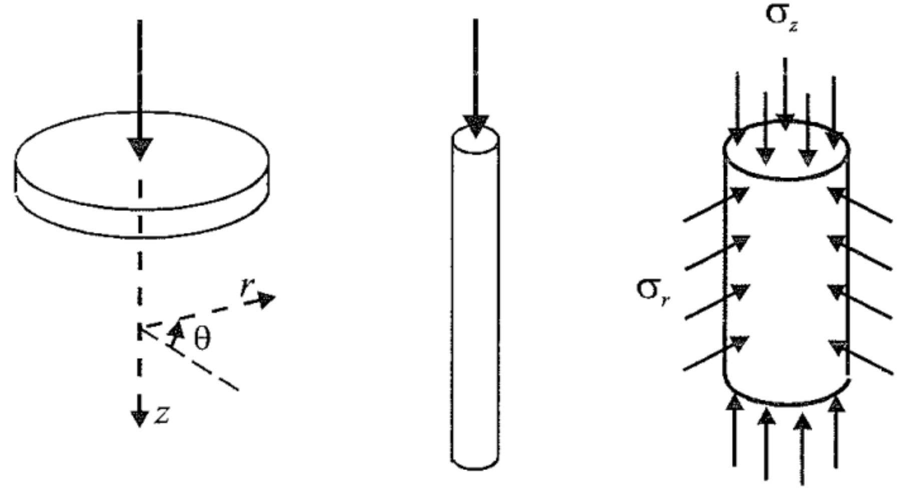
Settle 3D

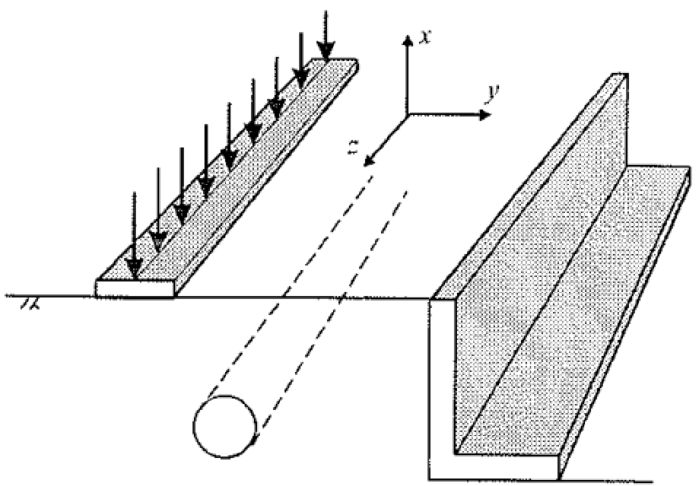


Cavity Expansion

Geometric Idealization

Plane Strain Conditions



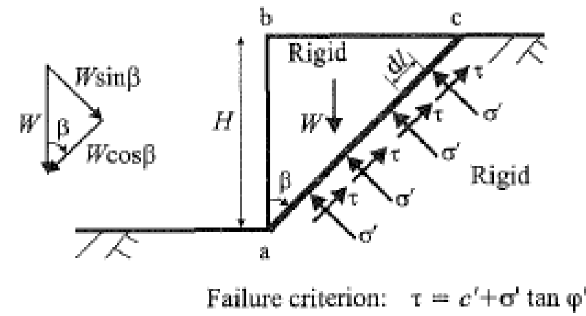


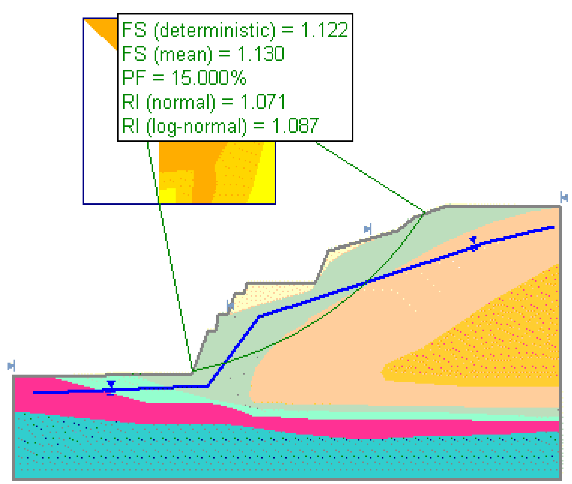
Axisymmetric Conditions

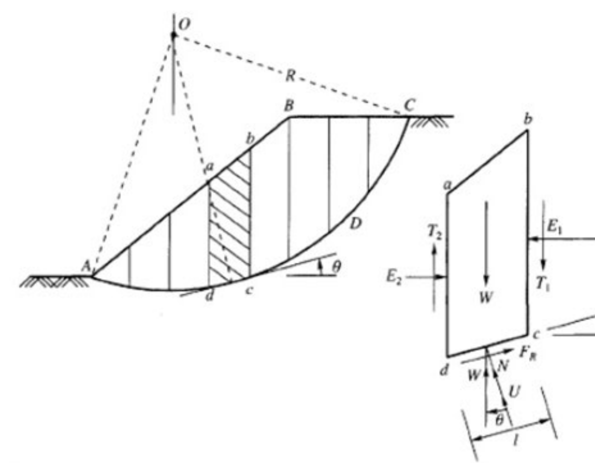
Conventional methods: Simple methods

Limit Equilibrium

- Coulomb's earth pressure
- Method of Slices



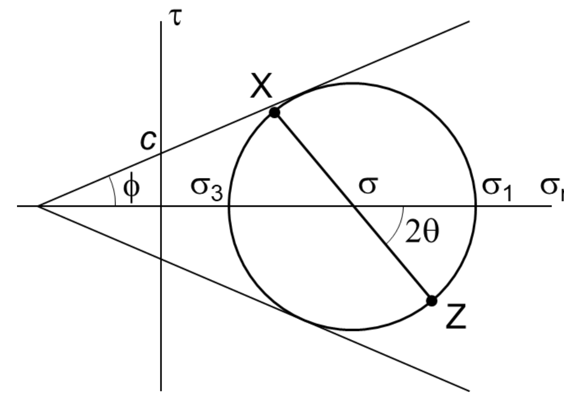




Conventional methods: Simple methods

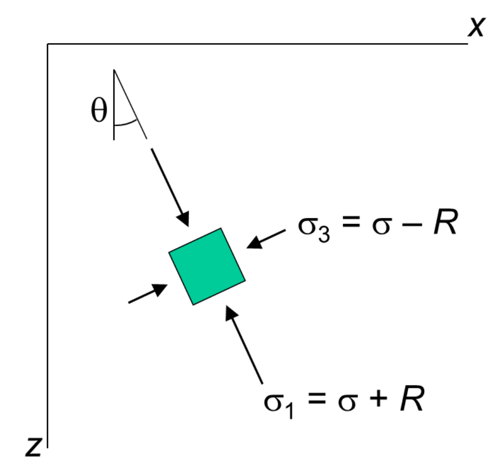
Stress Fields

- Rankine Earth Pressure
- Bearing Capacity



$$R = c\cos\phi + \sigma\sin\phi$$

Solving Equilibrium differential Equations with a failure criterion to solve for stresses every where



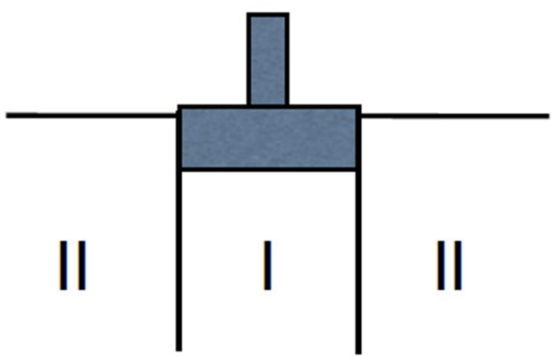
$$\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\tau_{xz}}{\partial z} = 0$$

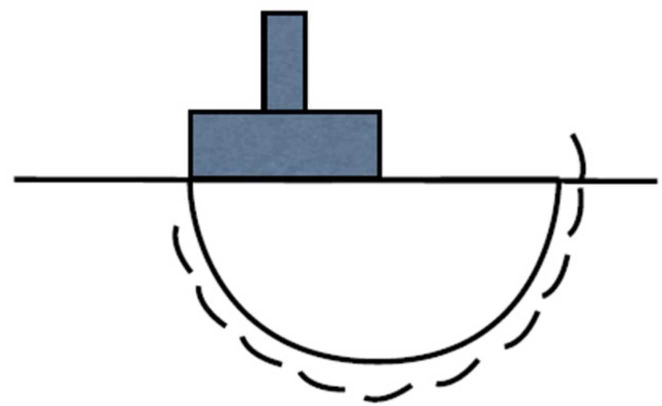
$$\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\sigma_{zz}}{\partial z} = \gamma$$

Conventional methods: Simple methods

Limit Analysis

The true solution of a problem lies between an upper bound which is an unsafe solution (ignores equilibrium) and a lower bound which is a too conservative solution (ignores compatibility)





Lower Bound Theorem

If a distribution of stresses can be found within the body which satisfies equilibrium at all points in the body and does not exceed yield criteria then exterior loads are lower bound loads to the true collapse loading conditions.

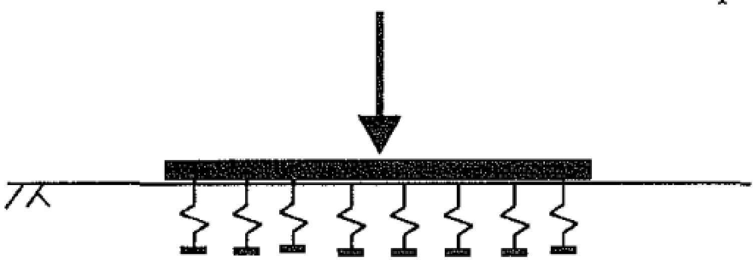
This is actually what we need: A Design value for the external loads that will always be less than the true collapse value

Upper Bound Theorem

If a distribution of velocity field (a mechanism of failure) can be found such that the rate of work done by the externally applied loads is larger than the energy dissipated internally by the material then exterior loads are upper bound loads to the true collapse loading conditions.

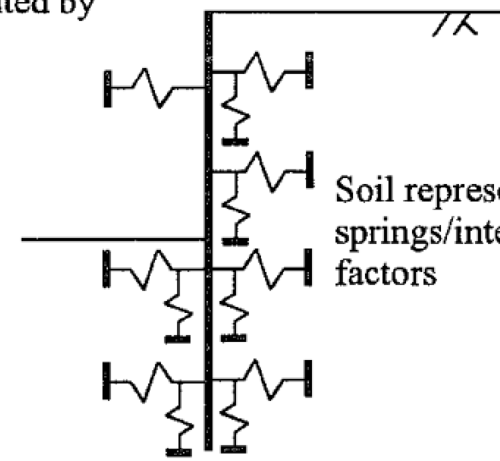
Numerical Methods

Beam Spring Approach



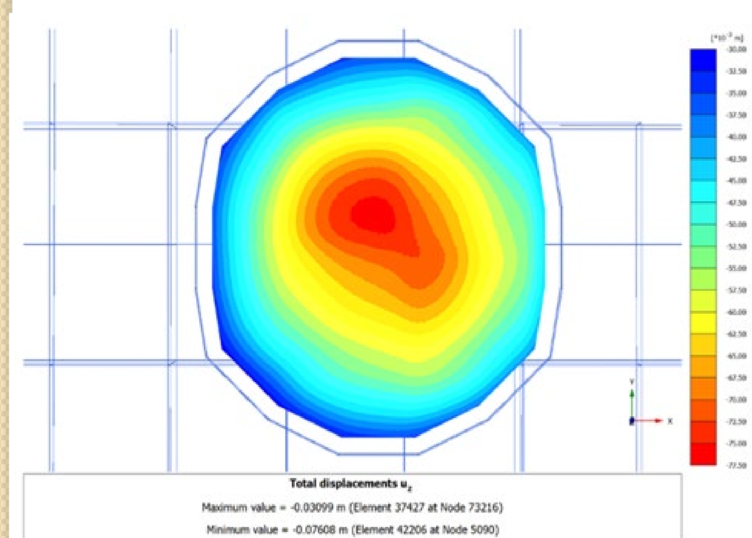
Soil represented by springs/interaction factors

Props/anchors represented by springs

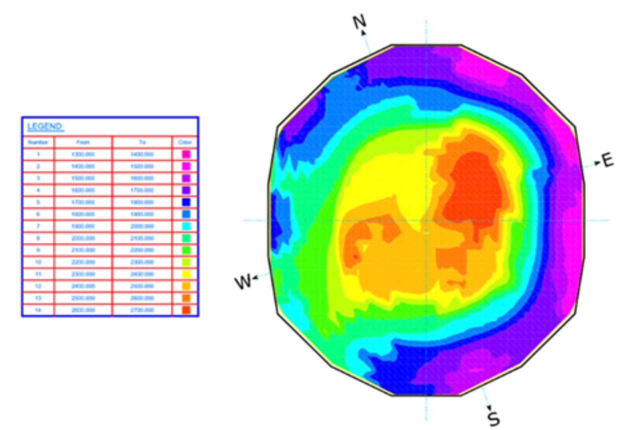


Soil represented by springs/interaction factors

Settlements



Subgrade Moduli



Full Numerical Analysis

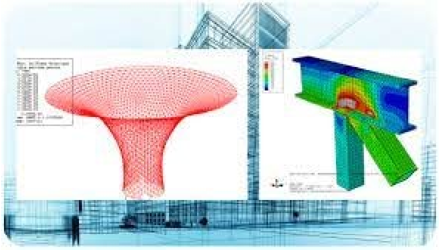
- Finite Difference

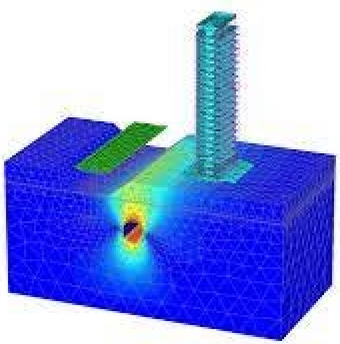
FLAC3D™

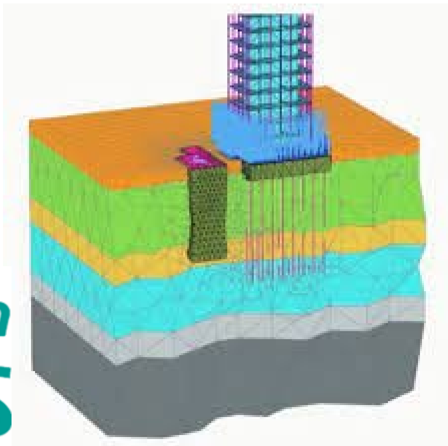
ITASCA™

- Finite Elements

PLAXIS



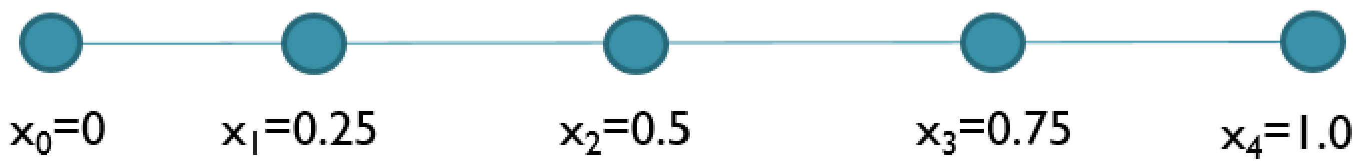




SIMULIA
ABAQUS

Finite Difference

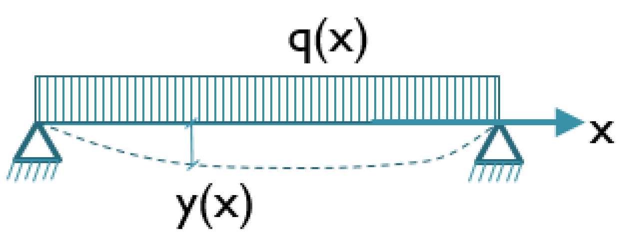
$$-EI\,y''(x) = M(x)$$



$y(0) = 0$? ? ? $y(1) = 0$

Using central difference formula for second derivative where i goes from 1 to 3

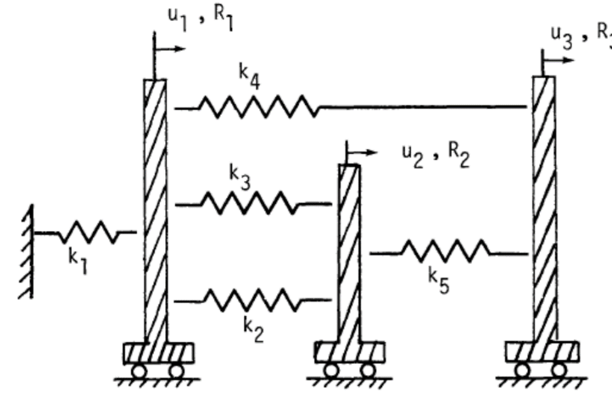
$$\frac{-y_{i-1} + 2y_i - y_{i+1}}{h^2} = \frac{1}{EI} M(x)$$



For a constant q(x) on a beam of L=1

$$M(x) = \frac{1}{2} q x - \frac{1}{2} q x^2$$

Finite Elements

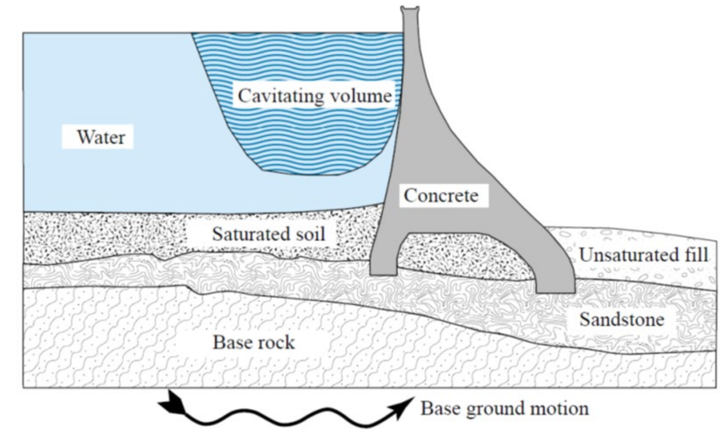


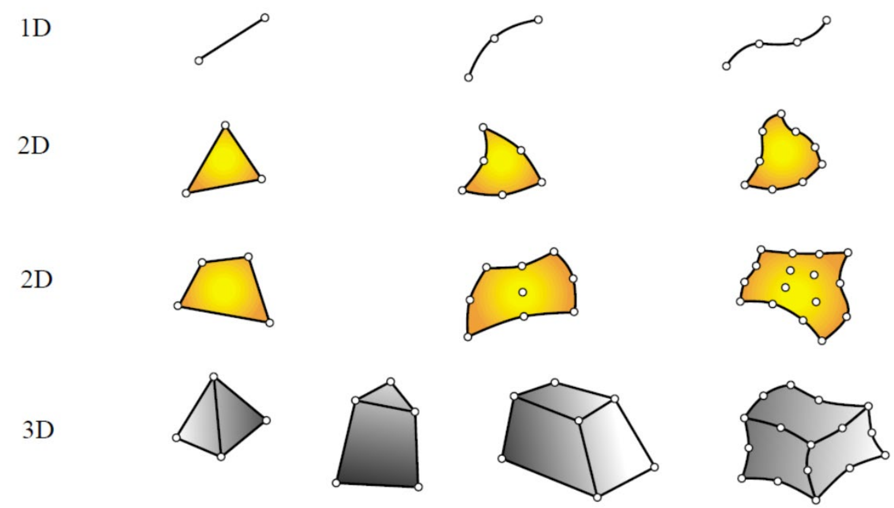
$$\underline{K} = \sum_{i=1}^{5} \underline{K}^{(i)}$$

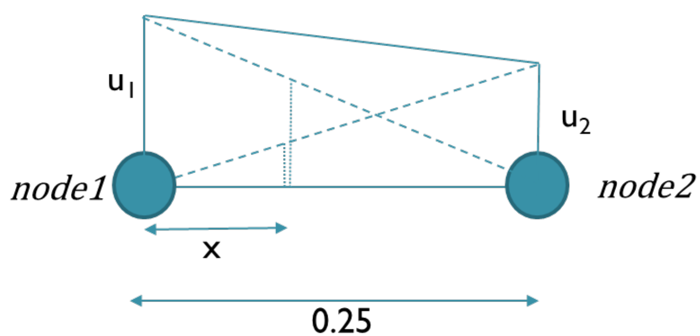
$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

$$\underline{K} = \begin{bmatrix} K_1 + K_2 + K_3 + K_4 & -K_2 - K_3 & -K_4 \\ -K_2 - K_3 & K_2 + K_3 + K_5 & -K_5 \\ -K_4 & -K_5 & K_4 + K_5 \end{bmatrix}$$

Analysis of a continuum



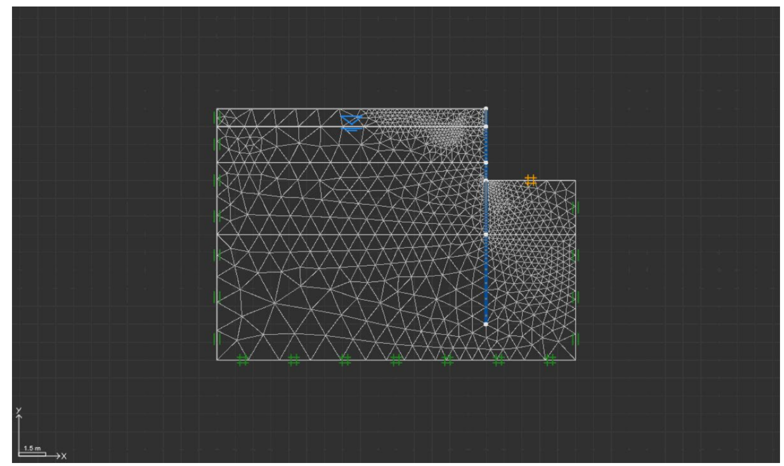




$$u_x = \left(\frac{0.25 - x}{0.25}\right) u_1 + \left(\frac{x}{0.25}\right) u_2$$

Finite Element Limit Analysis

- Scott Sloan (2010)
- Optum GE software
- Searching for the maximum lower bound and the minimum upper bound by linear programing



Solution Requirements

METHOD OF ANALYSIS		SOLUTION REQUIREMENTS				
		Equilibrium	Compatibility	Constitutive behaviour	Boundary conditions: Force	Boundary conditions: Disp
Closed form		S	S	Linear elastic	S	S
Limit equilibrium		S	NS	Rigid with a failure criterion	S	NS
Stress field		S	NS	Rigid with a failure criterion	S	NS
Limit analysis	Lower bound	S	NS	Ideal plasticity with associated flow rule	S	NS
	Upper bound	NS	S		NS	S
Beam-Spring approaches		S	S	Soil modelled by springs or elastic interaction factors	S	S
Full Numerical analysis		S	S	Any	S	S

S - Satisfied; NS - Not Satisfied

Comparison: Design Requirements

Table 2. Design requirements satisfied by the various methods of analysis

Method of analysis		Design requirements		
		Stability	Movements	Adjacent structures
Closed form (linear-elastic)		No	Yes	Yes
Limit equilibrium		Yes	No	No
Stress field		Yes	No	No
Limit analysis	Lower bound	Yes	No	No
	Upper bound	Yes	Crude estimate	No
Full numerical analysis		Yes	Yes	Yes

Potts, D. M. (2003). Numerical analysis: a virtual dream or practical reality?. *Géotechnique*, *53*(6), 535-573.

Advantages of Numerical Analysis

1- Complicated and non-linear soil behavior (effect of time dependence, pore pressure migration)

2- Construction sequence

3- Complicated soil structure interaction problems (adjacent structures, interconnection with structure components)

Challenges of Numerical Analysis

1- You have to be familiar with the constitutive model

2- You have to be familiar with the algorithms used to solve non-linear problems.

3- You have to be familiar with the software and how components are defined and the interaction between them.

Thank you